Module 5

Applied Mathematics 30

SINUSOIDAL DATA

Learning Technologies Branch

Alberta LEARNING

Module

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Module 5: Sinusoidal Data
Student Module Booklet
Learning Technologies Branch
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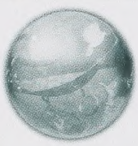
This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	



You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome

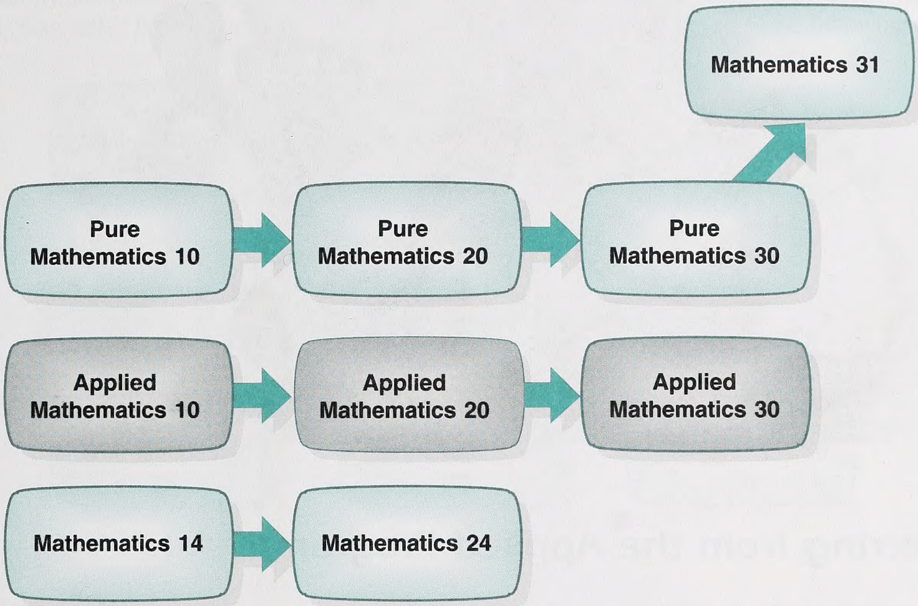
Applied Mathematics 30

Applied Mathematics 30 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

Contents

Introduction to Applied Mathematics 30

Applied Mathematics 30 is the third course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read any of the documents under the heading "New Senior High School Mathematics Update/Post-Secondary Studies Update" at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 30, it is recommended that you talk with a school counsellor about your career plans.

Transferring from the Applied Program

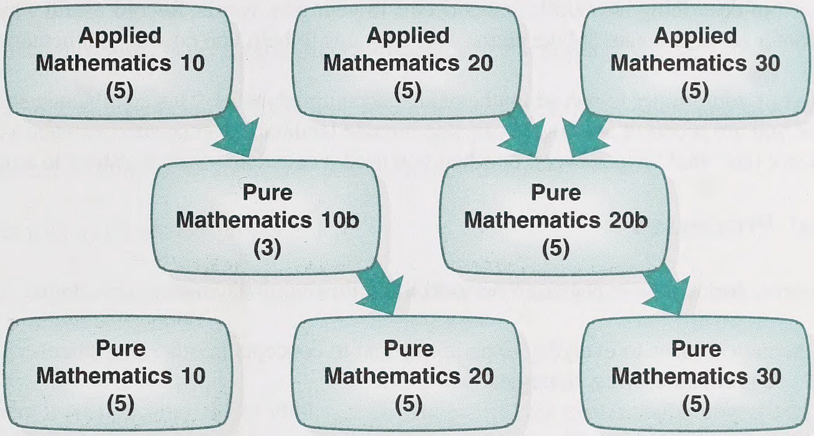
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectations • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

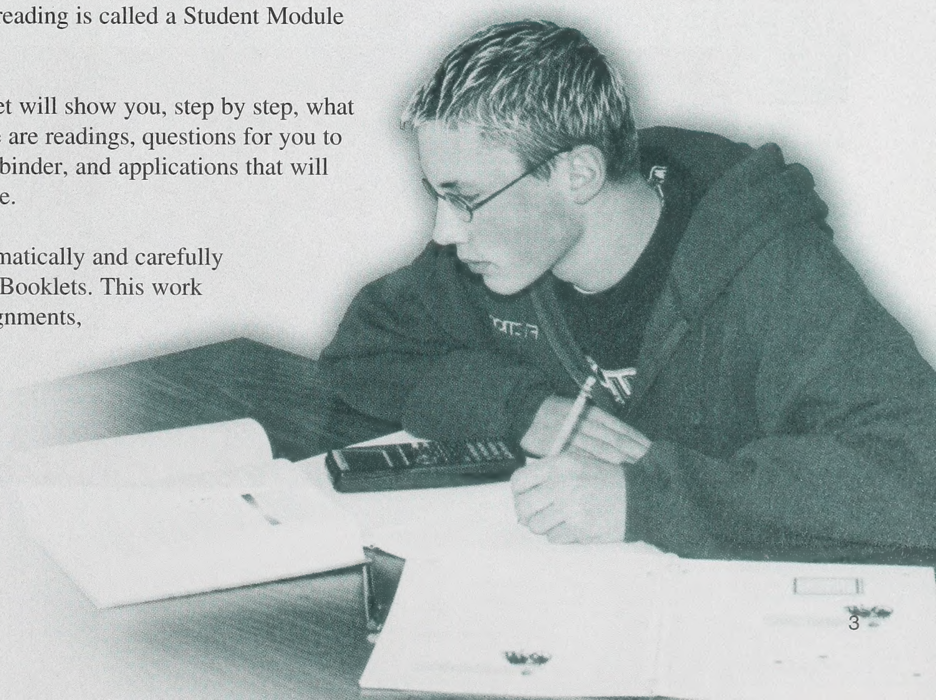


Strategies for Completing Applied Mathematics 30

For each module in Applied Mathematics 30, there is a Student Module Booklet and accompanying Assignment Booklets. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.

- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.

- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words. Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 30 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.

Resources You Will Need

In addition to the course materials for Applied Mathematics 30, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 12 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)

- the *Addison-Wesley Applied Mathematics 12 Project Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)

- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser

- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure

- a mathematical instrument set (compass, protractor, and triangles)

- a computer with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel.*

- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-83	EL-9600C	Algebra FX 2.0	HP 39g†
TI-83 Plus	EL-9600*	CFX-9850 GA-Plus*	
TI-86	EL-9200*	CFX-9850 G*	
TI-89	EL-9300*	CFX-9800 G*	
TI-92*		FX-9700 series*	
TI-92 Plus			

*no longer commercially available
† The HP 39g calculator will remain on the approved list for the 2001–2003 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor.*

Many of the resources you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

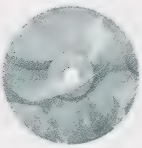
http://www.lrc.learning.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

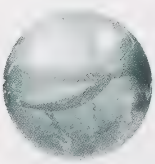
Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

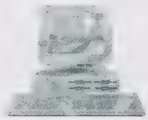
Refer to the textbook or the Project Book.



Refer to the Applied Mathematics 30 CD.



Explore the Internet.



Work with a computer.

Contact your teacher for additional information.

Complete specified questions in the Assignment Booklet.

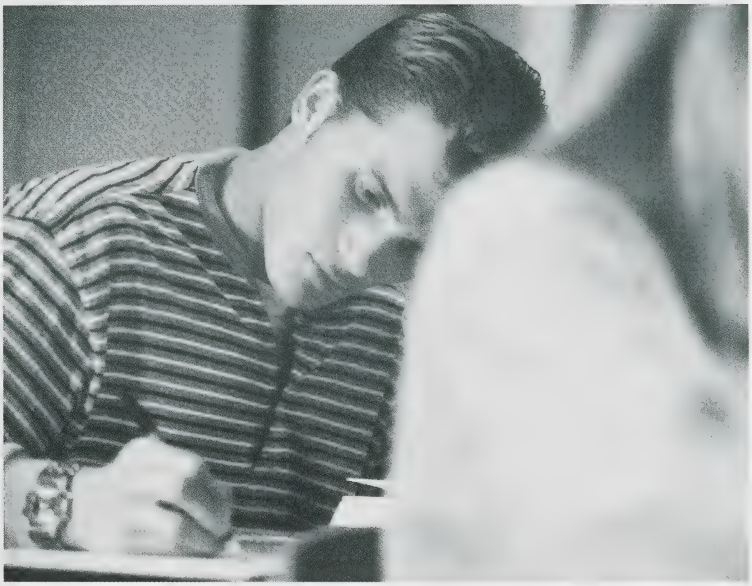
Remember: Any Internet website address given in this module is subject to change.

Where Can I Obtain Diploma Examination Information?

Alberta students will write a diploma examination at the end of the course. Alberta Learning provides several documents to help students prepare for this examination. These documents are found under the heading "Diploma Examinations" at the following Alberta Learning website:

http://www.learning.gov.ab.ca/k_12/testing

Information like course expectations, the makeup of the diploma examination, keyed copies of previous examinations, preparation guides, and calculator policies are available to students at this site.



Each year, in February and September, Alberta Learning provides teachers with information on a **student project**, which teachers **may** use as part of your overall assessment. Information to students will also be posted on the Alberta Learning website. Check with your teacher to determine what you will be expected to do. Be aware that one of the diploma examination's written-response questions will deal with elements of this project and is worth 10% of your diploma examination mark.

You should take advantage of the many sources of information about Applied Mathematics 30. Your success depends on your understanding of course expectations and evaluation procedures. Work closely with your teacher and do not hesitate to ask questions.

Remember, take the initiative to find out all you can about Applied Mathematics 30.

MODULE OVERVIEW

Throughout history, the full moon has kindled human imagination. The harvest moon—the full moon closest to the first day of autumn—fills the evening sky, bathing the landscape in brilliant moonlight and marking the passage of summer. The next full moon, or hunter's moon, does not appear as early in the evening. The phases of the moon recur at regular intervals, about every "month" or $29\frac{1}{2}$ days.

Did you know that Earth and the Moon form a system? Did you know that every $27\frac{1}{3}$ days they both revolve about the system's centre of mass, located a little more than 4500 km from the centre of Earth? It is this centre of mass that traces an elliptical path around the Sun. Both Earth and the Moon crisscross this orbit on their respective journeys.

Periodic motion—motion that recurs at regular intervals, like the motion of the Moon or the motion of Earth—can be modelled using trigonometric functions.

In this module, you will examine angular measure and sinusoidal functions. You will explore the graphs of the sine function, and you will apply the sine function to model periodic events.

For information about the moon's rotation and other periodic events, view the segment *Periodic Events* on the Applied Mathematics 30 CD.

Assessment

Accompanying this Student Module Booklet are two Assignment Booklets. Your grading in this module will be based upon the assignments you submit for assessment. The mark distribution is as follows:

Assignment Booklet 5A	
Activities 1 to 4 Assignment	**75 marks**
Assignment Booklet 5B	
Activity 5 Assignment	**40 marks**
Module Review Assignment	**50 marks**
Module Project	**40 marks**
TOTAL	**205 marks**

Remember that Activities 1 to 5 in this Student Module Booklet will prepare you for completing the module project and the module assignments. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Module Review provides a review of the module and an enrichment activity. You may choose to do some or all of the questions in the Module Review. Again, you should compare your answers with the suggested answers provided in the Appendix.

Beginning the Project

Your teacher may not require you to complete all the projects provided in this Applied Mathematics 30 course. Contact your teacher and check whether you need to complete the module project, Angle of Elevation of the Sun, as part of your assessment.

The sun is the brightest object in the Earth's sky. The sun rises and sets each day. Although the sunrise and sunset times are not the same day after day, the sunrise and sunset times do repeat over the period of a year. The sunrise in a particular location on a particular day in January is the same year after year. This repeating pattern is called a periodic event. Likewise, the angle of elevation of the sun is a periodic event and can be represented using a sinusoidal curve.

Your project for Module 5 is finding an equation for the angle of elevation of the sun. This project involves learning about periodic data, analysing periodic data, collecting data, and turning the data into a reasonable equation. You will discover how to use your graphing calculator to analyse the data and to find an equation of best fit from your data. You will then use this equation to make predictions about sun-related events.

Turn to page 208 of the textbook and read “Analyzing Periodic Data.” Answer the questions posed, and store them in the project section of your mathematics binder.

Page 208 of the textbook lists the Internet address of Addison Wesley Longman Ltd. This website has links to several sites you may find helpful in researching for this project.

As you work through Activities 1 to 5, continue to research ideas that will help you complete the project. You will find the concepts and exercises presented in this module, such as sinusoidal regression, particularly useful in completing the project.

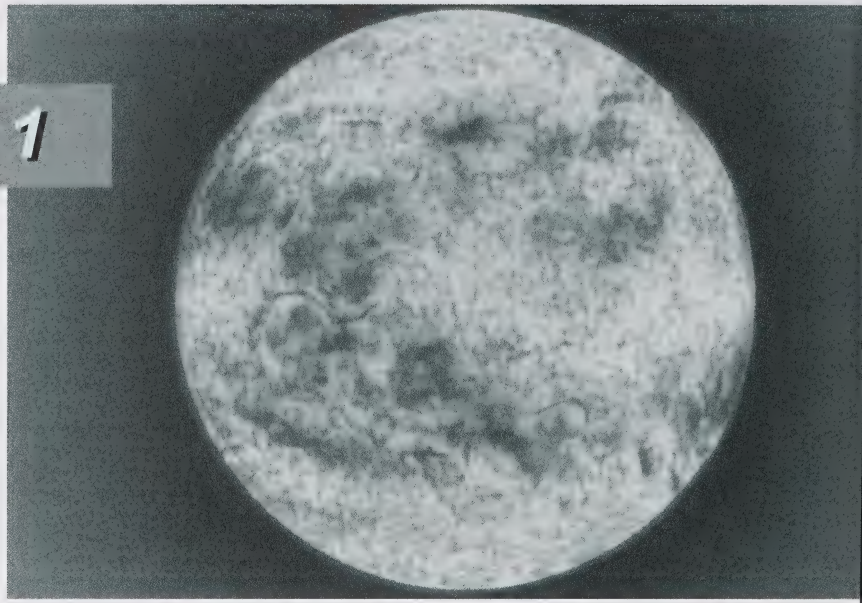
Turn to pages 220 and 221 of the textbook and study the data-acquisition requirements for the module project. You should be working on this part of the project at the same time as you are working through Activities 1 to 5.

> **WARNING:** Do not look at the sun, even with dark or smoked glasses. Do not look at the sun with binoculars or with a telescope. You can do irreparable damage to your eyes!

Complete exercises 1 and 2 of “Angle of Elevation of the Sun” on page 220 of the textbook. Store your responses in the project section of your mathematics binder.

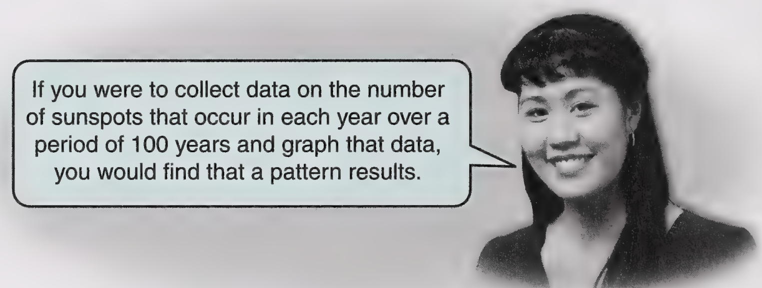
You will be given more direction on how to complete this project later in this module. In the meantime, feel free to discuss your project with your study partner or a family member. Remember, the work on the project you submit must be your own.

ACTIVITY 1



Collecting and Plotting Periodic Data

Astronomers have been studying sunspots for centuries. Sunspots are the dark spots that occur on the surface of the sun. These dark spots are due to magnetic storms and appear dark because the temperature of that region is lower than that of the surrounding surface The number of sunspots that occur in any year seems to occur in cycles.

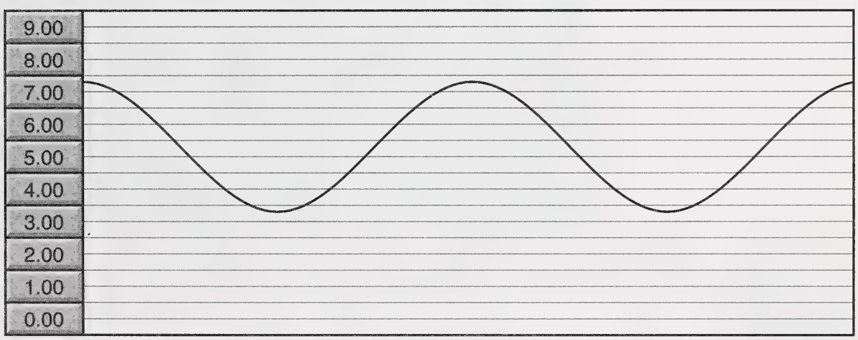


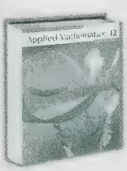
In this activity you will analyse various sources of collected data.

When you see a block of numbers, like those given here, the patterns and nuances are very difficult to pick out.

7.77	7.73	7.68	7.62	7.53	7.43	7.32	7.18	7.05
6.90	6.75	6.58	6.42	6.25	6.07	5.90	5.72	5.55
5.37	5.20	5.03	4.87	4.70	4.55	4.42	4.28	4.17
4.07	3.97	3.90	3.83	3.80	3.77	3.77	3.78	3.83
3.88	3.95	4.02	4.12	4.22	4.32	4.43	4.55	4.67
4.78	4.90	5.02	5.13	5.27	5.38	5.50	5.62	5.73
5.87	5.98	6.12	6.25	6.37	6.50	6.63	6.77	6.90
7.03	7.17	7.28	7.40	7.50	7.60	7.67	7.72	7.77
7.77	7.77	7.73	7.68	7.62	7.52	7.42	7.30	7.17
7.03	6.88	6.72	6.57	6.40	6.22	6.05	5.87	5.70
5.52	5.33	5.17	5.00	4.83	4.68	4.53	4.40	4.27
4.15	4.05	3.95	3.88	3.83	3.78	3.77	3.77	3.80
3.83	3.88	3.95	4.03	4.13	4.23	4.33	4.45	4.57
4.68	4.80	4.92	5.03	5.17	5.28	5.40	5.52	5.63
5.77	5.88	6.00	6.13	6.27	6.40	6.52	6.67	6.80
6.93	7.05	7.18	7.30	7.42	7.52	7.60	7.68	7.73
7.77	7.77							

One way to determine if a pattern exists is to graph the data. The following graph represents the preceding information. In the graph, the pattern becomes much more clear.





Turn to page 210 of the textbook and examine the four graphs at the top of the page. Pay particular attention to any pattern you can see in each graph. Then read the discussion following these graphs. This discussion will remind you of the importance of graphs when analysing data.

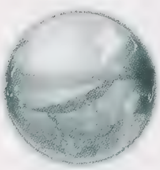
1. Complete exercises 1 to 6 of "Investigation 1: Patterns in a Machine: Clock Time" on pages 210 and 211 of the textbook.

2. Complete exercises 1 to 6 of “Investigation 2: Patterns in Nature: Times of Sunrise” on pages 211 and 212 of the textbook. Use the following table to complete the exercises.

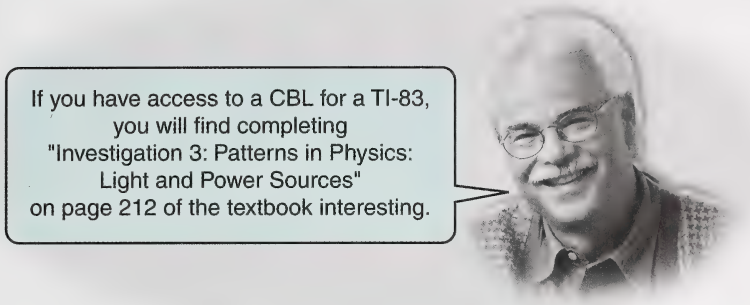
1998	Day Number	Time of Sunrise (A.M.)	1999	Day Number	Time of Sunrise (A.M.)
Jan. 15	15	8.53	Jan. 15	380	8.53
Feb. 15	46	7.83	Feb. 15	411	7.83
Mar. 15	74	6.88	Mar. 15	439	6.90
Apr. 15	105	5.78	Apr. 15	470	5.80
May 15	135	4.90	May 15	500	4.92
June 15	166	4.52	June 15	531	4.52
July 15	196	4.80	July 15	561	4.80
Aug. 15	227	5.50	Aug. 15	592	5.48
Sept. 15	258	6.27	Sept. 15	623	6.25
Oct. 15	288	7.02	Oct. 15	653	7.02
Nov. 15	319	7.87	Nov. 15	684	7.87
Dec. 15	349	8.52	Dec. 15	714	8.52

For exercise 6, visit the following website for sunrise times in Canada:

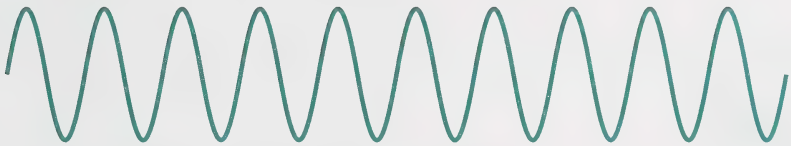
http://www.hia.nrc.ca/services/sunmoon/sunmoon.html



Compare your responses with the suggested answers in the Appendix, Activity 1, pages 56–59.

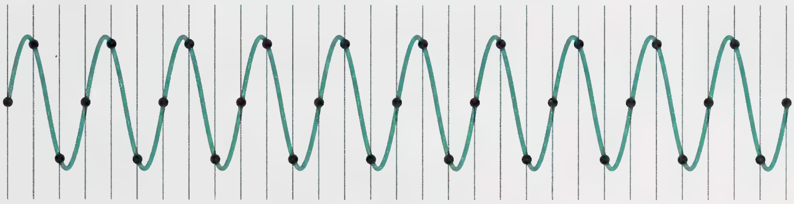


When you listen to your favourite CD, you are listening to data sampled over 44 000 times per second. This frequency was chosen after very careful consideration of sampling theory and studies of human hearing. Here is a diagram showing ten cycles of a sine wave.



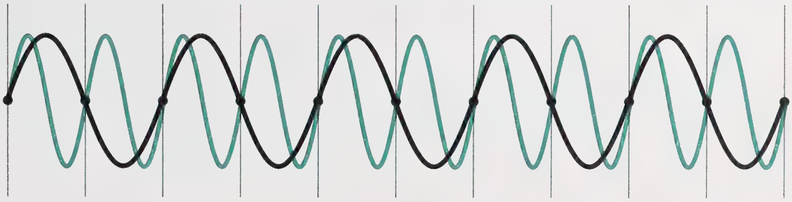
If this wave is sampled at more than twice its frequency, meaning more than two points are known in each cycle, the sine wave can be reconstructed from the data. Refer to the following diagram.

Sampled 3 Times Its Frequency

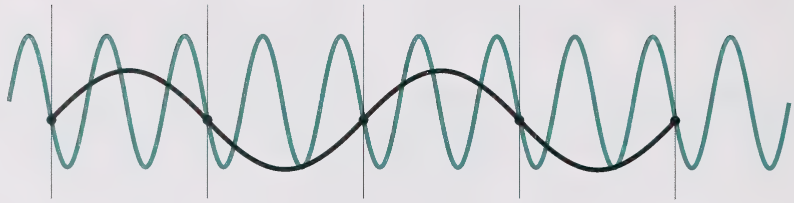


If the wave is sampled at less than twice its frequency, the original sine wave cannot be reconstructed from the data. Each of the following diagrams show other sine waves that could be constructed by not having enough reference points.

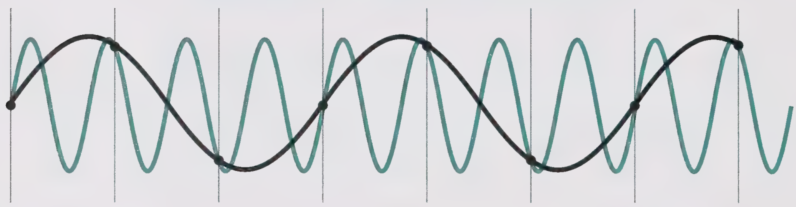
Sampled 1 Times Its Frequency

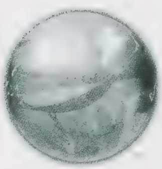


Sampled 0.5 Times Its Frequency



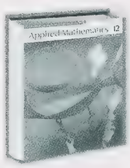
Sampled $\frac{3}{4}$ Times Its Frequency



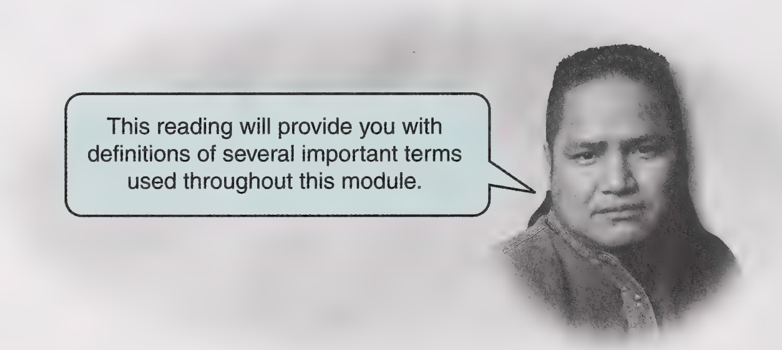


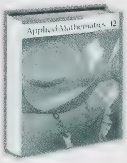
Analysing sampled data can lead to mistakes if the data collection is not done carefully. You have to watch the types of data you graph and be sure your analysis doesn't fall prey to these kinds of problems. If you are interested in sampling theory, you will want to learn about the Nyquist Theorem, which the preceding samples illustrate. The following websites give more information about sampling theory:

- **http://www.howstuffworks.com/analog-digital.htm**
- **http://www.efunda.com/designstandards/sensors/methods/DSP_nyquist.cfm**
- **http://www.bores.com/courses/intro/basics/1_antia.htm**



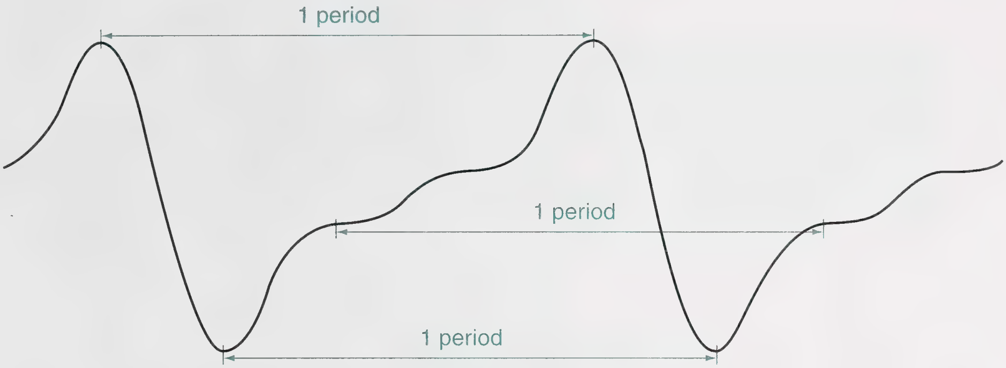
Investigations 1 and 2 involved natural data that repeated regularly over a fairly long time period. Turn to pages 212 and 213 and read the material following Investigation 3 up to Example 1.





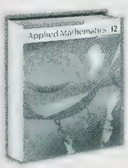
Now, turn to pages 213 and 214 of the textbook and work through "Example 1: Temperature." In this example you will see how graphs are used to analyse periodic data.

Did you notice how to measure the period of a **cyclic** or **periodic** graph? You measure from one identifiable feature to the same feature in the next period. Refer to the following diagram.



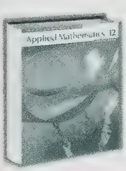
You will often use a particular point in the cycle to make taking measurements simpler. The following are examples of points you could choose:

- highest points
- lowest points
- median points



3. Answer exercises 1 and 4 of "Exercises: Checking Your Skills" on pages 216 to 218 of the textbook.

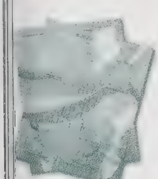
Compare your responses with the suggested answers in the Appendix, Activity 1, page 60.



Turn to page 215 of the textbook and work through "Example 2: Ferris Wheel." Here, you will see another example of using graphs to analyse periodic data.

4. Answer exercises 1, 2, and 3 of "Discussing the Ideas" on page 216 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 60.



Turn to page 1 of Assignment Booklet 5A and answer questions 1 and 2.

The following example shows how you can estimate the sunrise times of a city given the sunrise times of two other cities.

Example

The following table shows the sunrise times in Yellowknife, NWT, and in Havana, Cuba.

Sunrise Times in 2001

City	Mar. 21	Jun. 21	Sep. 21	Dec. 21
Yellowknife, NWT	6:34	2:40	6:19	10:07
Havana, Cuba	6:32	5:45	6:18	7:06

a. On the same grid, sketch the graphs of the sunrise times in Yellowknife and in Havana.

b. Winnipeg, Manitoba, lies between Yellowknife and Havana. On the same grid, sketch another graph to estimate the sunrise times in Winnipeg throughout the year.

Solution

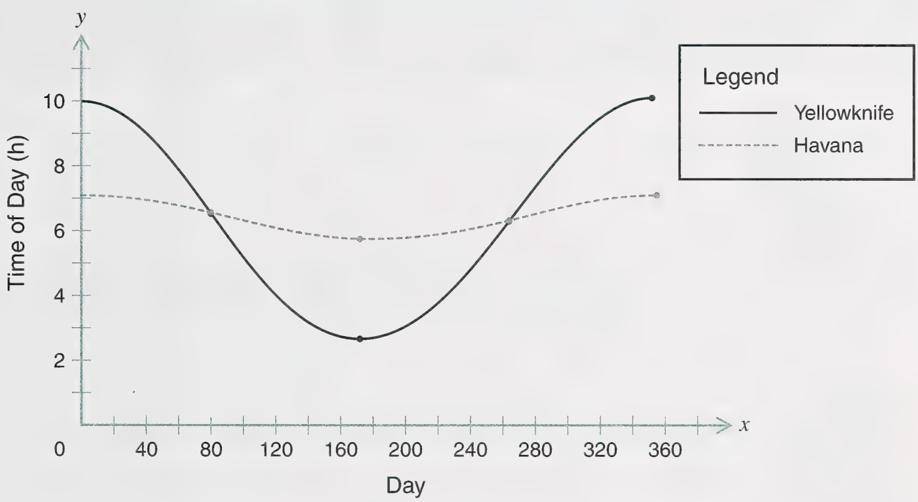
a. The following table shows the data converted to the day of the year and the times of day converted to hours.

Sunrise Times in 2001

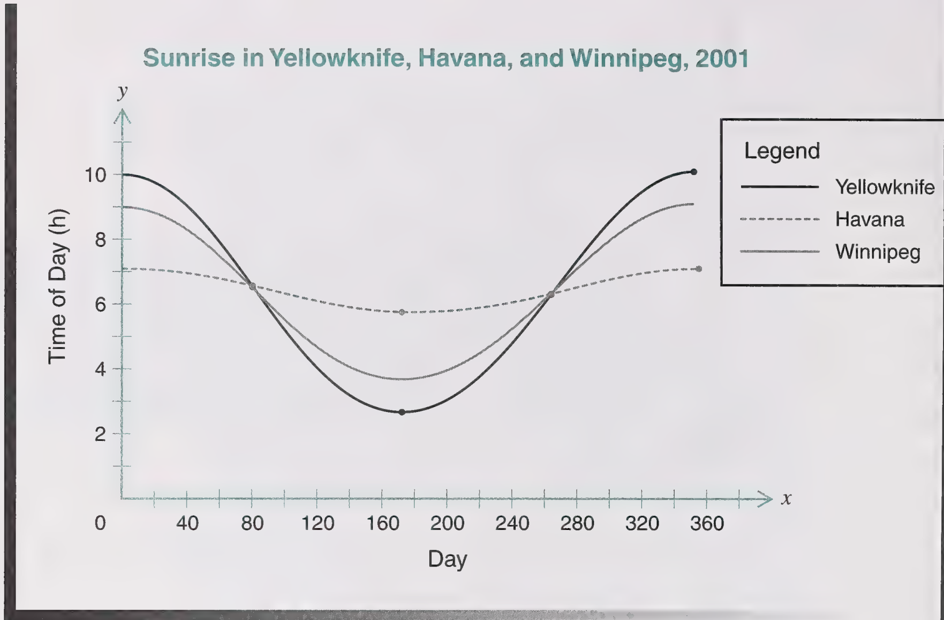
City	Day 80	Day 172	Day 264	Day 355
Yellowknife, NWT	6.57	2.67	6.32	10.12
Havana, Cuba	6.53	5.75	6.30	7.10

The graphs look like the following.

Sunrise in Yellowknife and Havana, 2001



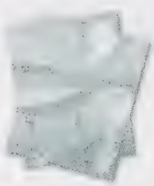
b. Because Winnipeg lies about a third of the distance from Yellowknife to Havana (north to south only), the graph of sunrise times in Winnipeg should also lie about a third of the distance between the graphs of Yellowknife and Havana and overlap at the spring and fall equinox.



5. Turn to pages 217 to 219 of the textbook and answer the following.

a. exercises 3, 5, and 7 of "Exercises: Checking Your Skills"
b. exercise 8 of "Exercises: Extending Your Thinking"

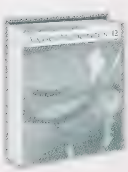
Compare your responses with the suggested answers in the Appendix, Activity 1, pages 61–64.



Turn to pages 1 and 2 of Assignment Booklet 5A and answer question 3.

Looking Back

In this activity you studied periodic and sinusoidal events. You graphed data of periodi events and analysed them. You then determined the period of repetitive data and estimated values for parts of events that were not in the recorded data.



6. Turn to page 219 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 1, page 64.

ACTIVITY 2

Radian Measure and Sine Curves

The United Nations started out of the determination to keep the world from being engulfed in another world war. Its headquarters in New York is a symbol of the world's attempts to use reason, not warfare. There are hundreds of countries and many languages and dialects; it's a wonder people are able to communicate with each other at all. One idea has to be translated into hundreds of different languages for the representatives to understand what is being said.

Mathematics, too, has ideas that can be expressed in many different terms. In earlier grades you measured angles in degrees. In higher mathematics, angles are often measured in radians. In this activity you will examine **radian measure** and how radians and degrees are related. This will be useful to you as you study the sine function and how it can be used in analysing periodic data.

Turn to page 222 of the textbook and read the introductory paragraphs of Tutorial 5.2, "Radian Measure and Sine Curves."

1. Complete exercises 1 to 10 of "Investigation 1: What Is a Radian?" on pages 222 and 223 of the textbook.

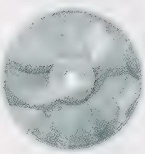
Compare your responses with the suggested answers in the Appendix, Activity 2, pages 65–66.

Turn to page 2 of Assignment Booklet 5A and answer question 4.

Turn to page 223 of the textbook and read the section between Investigation 1 and Investigation 2. Here, you will see how radian measure and degree measure are related.

Note the convention that an angle measurement is in radians unless some other measurement system is given. This means the angle measures in the first column are in radians, while those in the second column are not.

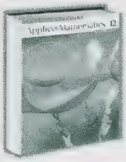
Radian Measure for Angles	Degree Measure for Angles
$\angle A = 3$	$\angle A = 172°$
$\angle C = 2.7$	$\angle A = 155°$
$\angle MNO = \frac{3\pi}{7}$	$\angle MNO = 77°$
$\angle H = 0.75$	$\angle H = 43°$



2. View the segment *Radian* on the Applied Mathematics 30 CD, and write a description of what is happening in the animation. Explain how the animation relates to radian measure for angles.

Compare your response with the suggested answer in the Appendix, Activity 2, page 66.

Example

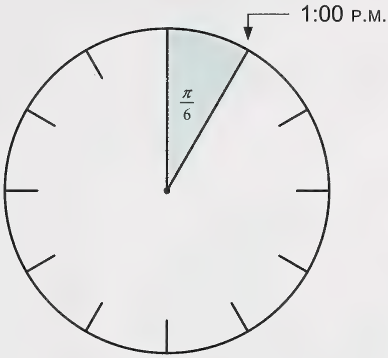


Revisit “Investigation 1: Patterns in a Machine: Clock Time” on pages 210 and 211 of the textbook, and answer questions 2 and 3 using radian measure rather than degrees.

Solution

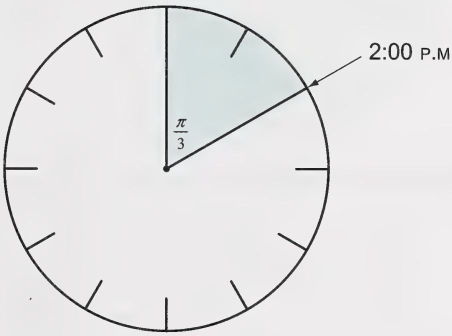
2. The hour hand passes through an angle of $2\pi \div 12 = \frac{\pi}{6}$ in one hour.

3. **a.**



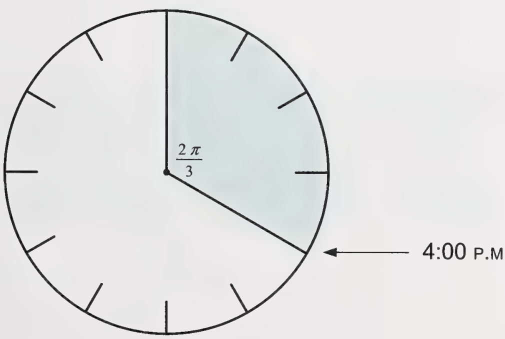
$$\frac{\pi}{6}$$

b.



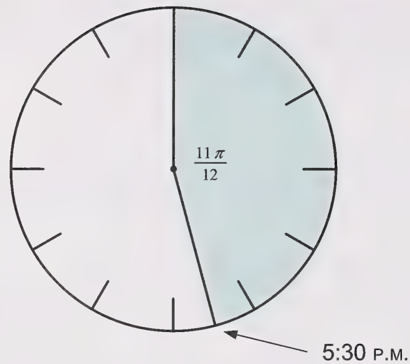
$$2 \times \frac{\pi}{6} = \frac{2\pi}{6}$$
$$= \frac{\pi}{3}$$

c.



$$4 \times \frac{\pi}{6} = \frac{4\pi}{6}$$
$$= \frac{2\pi}{3}$$

d.



$$5.5 \times \frac{\pi}{6} = \frac{5.5\pi}{6}$$
$$= \frac{11\pi}{12}$$

e.

$$10.2 \times \frac{\pi}{6} = \frac{10.2\pi}{6}$$
$$= \frac{102\pi}{60}$$
$$= \frac{17\pi}{10}$$

Turn to page 2 of Assignment Booklet 5A and answer question 5.

The next investigation will show you some of the important features of the sine graph. You will work with the terms **amplitude**, **period**, **minimum value**, and **maximum value**.

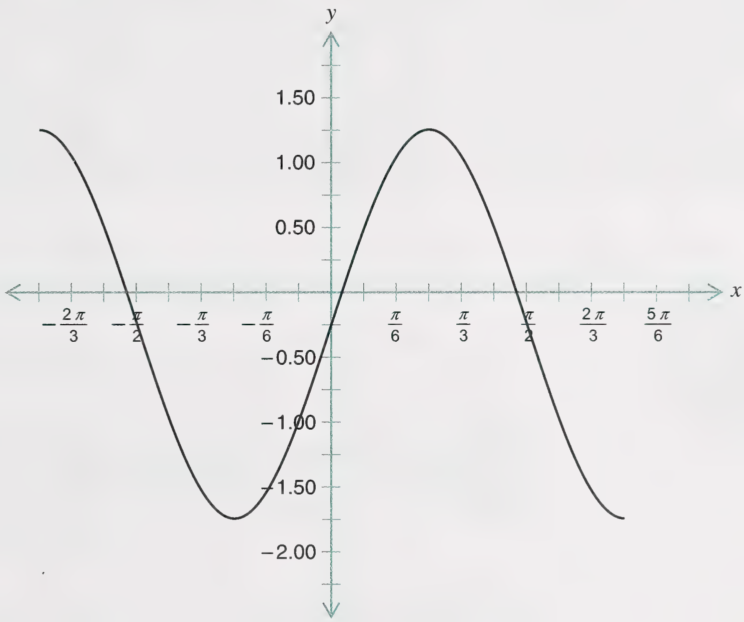
3. Turn to pages 223 and 224 of the textbook and complete exercises 1 to 8 of "Investigation 2: The Characteristics of $y = \sin x$."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 66–69.

The sinusoidal function can be shifted up and down as well as left or right. Work through the example that follows.

Example

What is the amplitude and period of the following sinusoidal function?



Solution

The amplitude can be found from the maximum and minimum values. The graph shows that the maximum value is 1.25 and the minimum value is -1.75.

$$\begin{aligned}\therefore\ \text{Amplitude} &= \frac{\text{Maximum value} - \text{Minimum value}}{2}\\ &= \frac{1.25-(-1.75)}{2}\\ &= \frac{3}{2}\\ &= 1.5\end{aligned}$$

The period is determined by finding the distance between two identical points on two successive cycles of the function. Thus, the period can be found from the x-values of two successive maximums or two successive minimums. You only need to use one of these methods, but both are shown anyway.

Method 1: Using Maximum Values

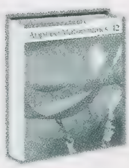
The x-coordinates of two successive maximum points of the given function are $-\frac{3\pi}{4}$ and $\frac{\pi}{4}$.

$$\begin{aligned} \therefore \text{ Period} &= \frac{\pi}{4} - \left(-\frac{3\pi}{4}\right) \\ &= \frac{\pi}{4} + \frac{3\pi}{4} \\ &= \frac{4\pi}{4} \\ &= \pi \end{aligned}$$

Method 2: Using Minimum Values

The x-coordinates of two successive minimum points of the given function are $-\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

$$\begin{aligned} \therefore \text{ Period} &= \frac{3\pi}{4} - \left(-\frac{\pi}{4}\right) \\ &= \frac{3\pi}{4} + \frac{\pi}{4} \\ &= \frac{4\pi}{4} \\ &= \pi \end{aligned}$$



4. Turn to page 224 of the textbook and answer questions 1 to 4 of "Discussing the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 69–70.

Looking Back

In this activity you studied radian measure and the sine curve. You also examined the meaning of amplitude and period as they relate to sinusoidal curves.

5. Turn to page 224 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 2, page 71.

ACTIVITY 3

Fitting Sine Curves to Data

Millions of shoes are sold in North America every month, but how many of them actually fit the foot well? Many times people buy shoes because they look great, not because they are comfortable. Some news reports have put this figure as high as 85% of shoe buyers. Finding a shoe that looks great and fits well is something that takes perseverance and knowledge. The same can be said for fitting a mathematical equation to data. You have to consider what kind of equation to use, and you have to choose the coefficients carefully so the fit is good. Just like buying a pair of shoes, it's not "one size fits all," at least not one size fits all well.

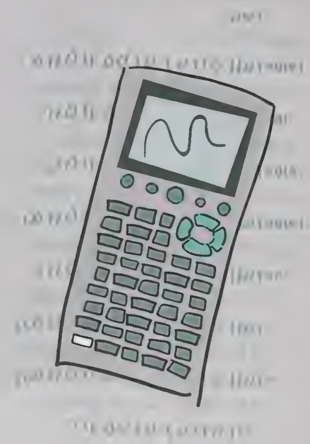
In this activity you will use the Sinusoidal Regression (SinReg) feature on your graphing calculator to fit sine curves to periodic data. You will have to use good judgment to make this work well.

The Sinusoidal Regression feature will work with no parameters following it, and this will often give usable results. Better results, however, generally occur when parameters are used. The parameters accepted are as follows:

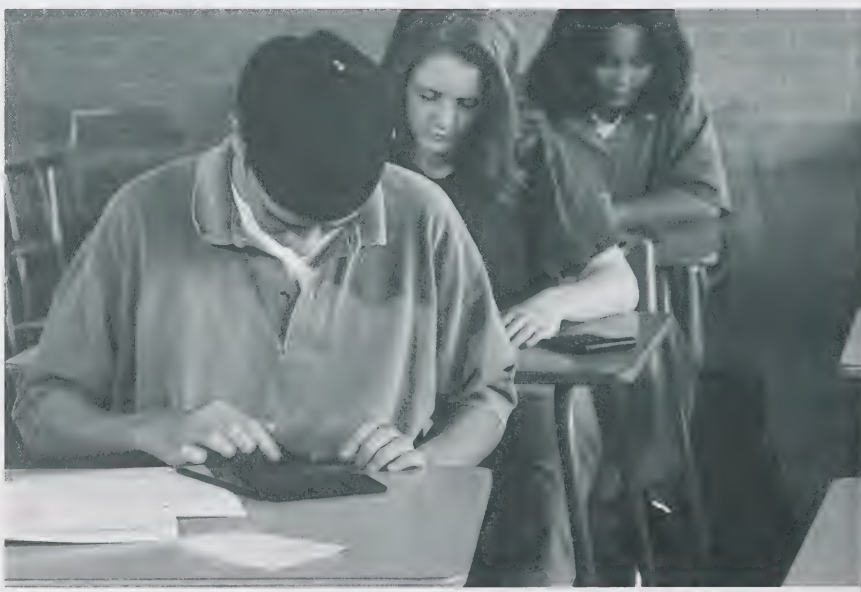
- a number between 1 and 16, which tells the calculator how many times to refine its estimates
- the list containing the independent data
- the list containing the dependent data
- an estimate for the period
- a place in the equation editor in which to store the resulting equation

Turn to page 225 of the textbook and read the introductory paragraphs of Tutorial 5.3, "Fitting Sine Curves to Data."

When using the Sinusoidal Regression feature, follow these steps to obtain the best results:



- Enter the data into lists L1 and L2 (or others if appropriate).
- Ensure that plotting is turned on.
- Set the window width and height to include all of the data points.
- Plot the points to ensure they are entered correctly.
- Use the plot to estimate the period of the data.
- Use the Sinusoidal Regression (SinReg) feature to obtain an estimated best-fit curve.
- Graph the sine equation obtained with the data to check the fit.
- Polish the result by repeating the regression using a new estimate for the period and more iterations.

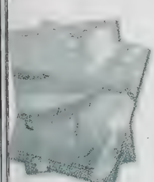


1. Using the following table, complete exercises 1 to 6 of "Investigation 1: Times of Sunrise" on pages 225 and 226 of the textbook. **Note:** Use an initial period of 365.25.

> Refer to "Utility 32: Using the TI-83 to Fit a Regression Equation to Periodic Data" on pages 361 and 362 for help with using the Sinusoidal Regression feature.

1998	Day Number	Time of Sunrise (A.M.)	1999	Day Number	Time of Sunrise (A.M.)
Jan. 15	15	8.53	Jan. 15	380	8.53
Feb. 15	46	7.83	Feb. 15	411	7.83
Mar. 15	74	6.88	Mar. 15	439	6.90
Apr. 15	105	5.78	Apr. 15	470	5.80
May 15	135	4.90	May 15	500	4.92
June 15	166	4.52	June 15	531	4.52
July 15	196	4.80	July 15	561	4.80
Aug. 15	227	5.50	Aug. 15	592	5.48
Sept. 15	258	6.27	Sept. 15	623	6.25
Oct. 15	288	7.02	Oct. 15	653	7.02
Nov. 15	319	7.87	Nov. 15	684	7.87
Dec. 15	349	8.52	Dec. 15	714	8.52

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 71–75.



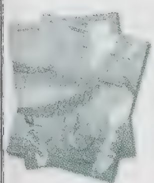
Turn to page 3 of Assignment Booklet 5A and answer question 6.

2. Using the following information, complete exercises 3, 4, and 5 of “Investigation 2: Determine the Frequency of a Musical Note” on page 226 of the textbook.

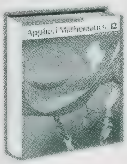
Time (s)	CBL Values	Time (s)	CBL Values
0.0002	127	0.0063	112
0.0004	149	0.0065	91
0.0007	168	0.0068	76
0.0009	183	0.0070	66
0.0011	191	0.0072	63
0.0013	191	0.0074	69
0.0015	184	0.0076	81
0.0017	170	0.0079	98
0.0020	151	0.0081	119
0.0022	130	0.0083	141
0.0024	108	0.0085	162
0.0026	88	0.0087	178
0.0028	73	0.0089	189
0.0031	65	0.0092	192
0.0033	64	0.0094	187
0.0035	70	0.0096	176
0.0037	84	0.0098	158
0.0039	102	0.0100	138
0.0041	123	0.0103	116
0.0044	145	0.0105	95
0.0046	165	0.0107	78

Time (s)	CBL Values	Time (s)	CBL Values
0.0048	181	0.0109	67
0.0050	190	0.0111	63
0.0052	191	0.0113	67
0.0055	186	0.0116	78
0.0057	173	0.0118	95
0.0059	155	0.0120	115
0.0061	134		

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 75–78.



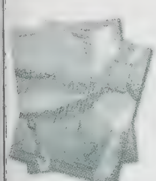
Turn to pages 3 and 4 of Assignment Booklet 5A and answer question 7.



3. Turn to page 229 of the textbook and answer exercise 1 of "Exercises: Checking Your Skills." **Note:** Use 365.25 as the initial period.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 78–80.

There are a large number of naturally occurring events that are sinusoidal. The rising and setting of the sun are only two of a multitude of natural phenomena that allow the fitting of a sinusoidal function to their data. In earlier mathematics courses, you studied some of the other regression functions in your graphing calculator. You need to keep in mind that there are several regression methods. When trying to find an equation to describe a data set, you do not always need to use SinReg.



Turn to page 4 of Assignment Booklet 5A and answer question 8.

Turn to page 227 of the textbook and read the paragraph preceding the Example. Then work through "Example: Sine Regression and Tides" on pages 227 and 228.

4. Answer exercises 3 and 4 of "Discussing the Ideas" on page 229 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, page 80.

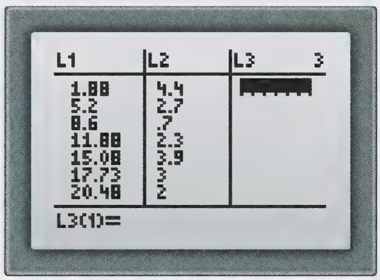
Turn to page 4 of Assignment Booklet 5A and answer question 9.

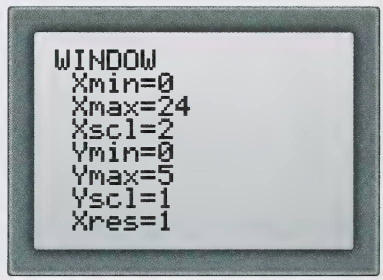
Example

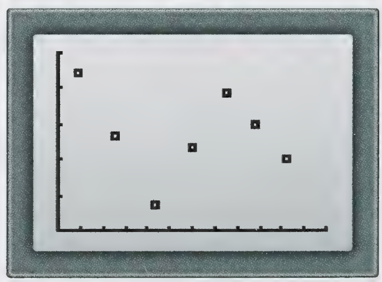
Answer exercise 3 of "Exercises: Checking Your Skills" on page 230 of the textbook.

Solution

3. a. Enter the time into list L1 and the depth into list L2; then graph the data.

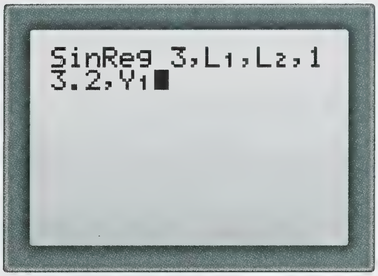


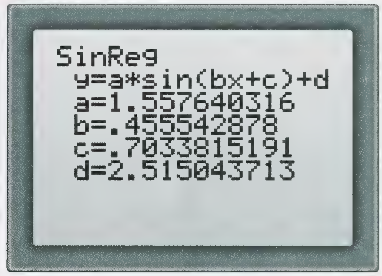


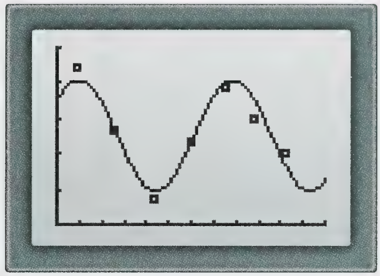


The data appears to have a period of approximately $15.08 - 1.88 = 13.2$ h.

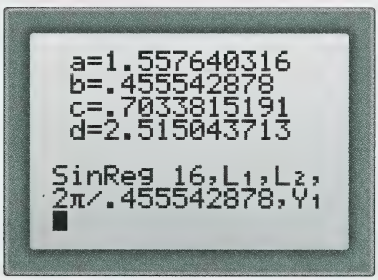
Determine and graph the initial regression equation.

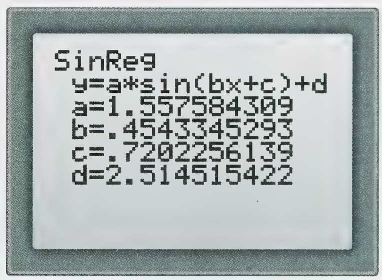






Obtain a better-fit equation and graph it.





Because the values differ only slightly from the initial regression, there is no need to obtain further better-fit equations.

Therefore, the sinusoidal regression equation is

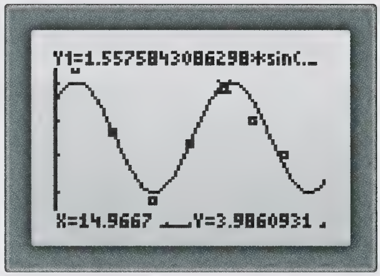
$y \doteq 1.558 \sin(0.454x + 0.720) + 2.515.$

b. $\dfrac{58}{60} \doteq 0.9667$

Therefore, 14:58 is approximately 14.9667 h.

Method 1: Using the Graph

Use the Value feature from the CALCULATE menu to determine the depth.

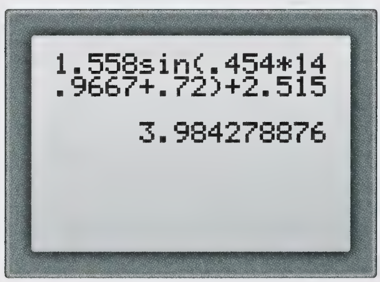


The depth of the water at 14:58 is about 3.99 m.

Method 2: Using the Regression Equation

Substitute 14.9667 for x into the regression equation.

$$\begin{aligned} y &\doteq 1.558 \sin(0.454x + 0.720) + 2.515 \\ &\doteq 1.558 \sin[0.454(14.9667) + 0.720] + 2.515 \\ &\doteq 3.984278876 \end{aligned}$$



The depth of the water is about 3.98 m.

c. The median depth of water corresponds to d in the regression equation. Therefore, the median depth of water is about 2.51 m.

d. The maximum is one amplitude above the median. Since a represents the amplitude, the maximum depth of water varies about 1.56 m above the median depth of water.

5. Answer exercises 4 and 5 of "Exercises: Checking Your Skills" on page 231 of the textbook. For exercise 5, use the following table.

Date (1999)	Day Number	Maximum Temperature (°C)
Jan. 1	1	−28.8
Feb. 1	32	−13.8
Mar. 1	60	−7.2
Apr. 1	91	3.2
May 1	121	12.8
June 1	152	20.4
July 1	182	17.8
Aug. 1	213	22.0
Sept. 1	244	13.0
Oct. 1	274	1.1
Nov. 1	305	0.6
Dec. 1	335	−13.2

6. Answer exercise 6 of "Exercises: Extending Your Thinking" on page 232 of the textbook. **Remember:** Turn on Plot 2 and Plot 3 using lists L1 and L3 and lists L1 and L4, respectively. Use different points for the second and third graphs. Also, for SinReg, save the equations in Y_2 and Y_3.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 80–91.

Project: Surviving on the Ocean's Shore

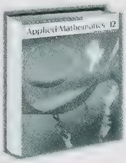
Now that you have had some experience with sinusoidal equations, you will do a proje that requires you to use sinusoidal functions to produce mathematical models to predict tide levels.

Turn to pages 80 to 82 of the Project Book and read "The Task" and "Background." Study the questions posed on page 82 regarding marine life in the inter-tidal zone. Kee these questions in mind as you do additional research on this topic.

You may wish to visit the following website for more information on inter-tidal life:

http://www.mit.edu/people/corrina/tidepool.html

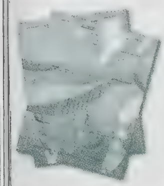
Here you will find a description of the variety of marine life that lives along the ocean' shore. You may also do your own Internet search by entering the words *animals* and *inter-tidal zone*. Also, your local library may have books on tides and marine life in the inter-tidal zone.



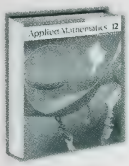
7. Answer the questions posed on page 82 of the Project Book.

8. Complete exercises 1, 2, and 3 of "Getting Started" on page 83 of the Project Book. **Note:** In exercise 3, use the Maximum and Minimum features from the CALCULATE menu to predict the time of high and low tides respectively.

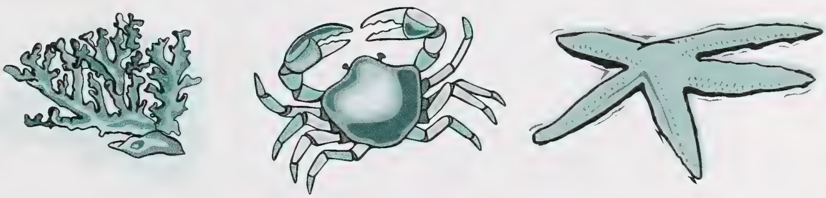
Compare your responses with the suggested answers in the Appendix, Activity 3, pages 91–93.



Turn to page 5 of Assignment Booklet 5A and answer question 10.

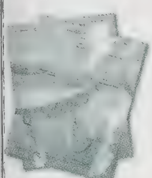


Turn to pages 84 and 85 of the Project Book and read "Project Presentation."



9. Describe the jobs of an oceanographer and marine biologist.

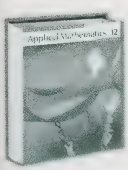
Compare your response with the suggested answer in the Appendix, Activity 3, page 93.



Turn to page 6 of Assignment Booklet 5A and answer question 11.

Looking Back

In this activity you studied how to find sinusoidal regression equations for periodic data. You then used this skill to complete the project, Surviving on the Ocean's Shore, relating to ocean tides and how changes in the water level can be represented mathematically.



10. Turn to page 232 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 3, page 93.

ACTIVITY 4

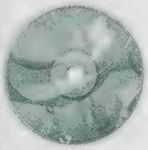
The Characteristics of $y=a\sin(bx+c)+d$

The characteristics that cattle breeders find significant in their animals depend on the breeders' ideas of what is important. Do they need animals that can look after themselve in rough terrain? Is it important that they not have trouble with heat or cold? Do they need animals to produce large quantities of milk?

For centuries, breeders have worked very hard to improve their animals. The characteristics that they have emphasized have related to their needs and circumstances. Breeds, such as Holsteins, Jerseys, and Guernseys, have been bred to produce milk. Oth breeds, such as Black Angus, Hereford, and Simmental, have been developed to produc meat. The Texas Longhorn, however, wasn't so much bred by humans as it was shaped by nature. This tough and adaptable breed survived the rigors of looking after themselve on the range almost as a wild creature.

In this activity you will look carefully at the characteristics of sinusoidal functions. You like cattle owners, will need to know what you are looking for and how to emphasize it. You will study how the characteristics of period, amplitude, horizontal shift, and vertica shift affect a sinusoidal equation.

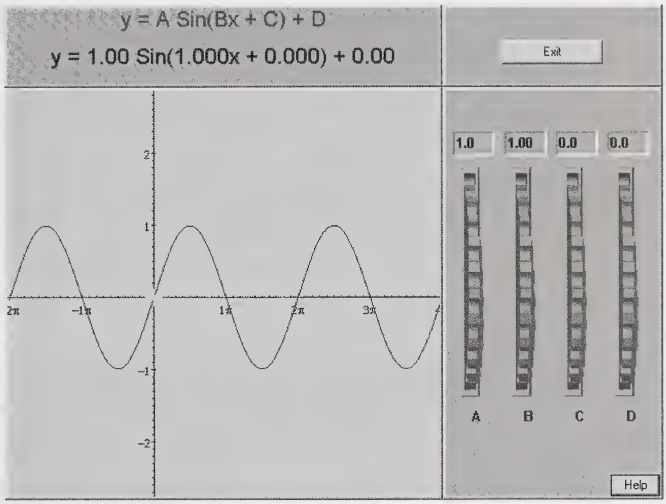
Turn to page 233 of the textbook and read the introductory paragraphs of Tutorial 5.4, "The Characteristics of $y=a\sin(bx+c)+d$."



Open the *Sine Explorer* located on the Applied Mathematics 30 CD. Try the following exercises to learn some of the features of the program and to learn about the coefficients in a general sine equation.

> **Note:** If you are unable to run the explorer, do not do exercises 1 and 2.

1. Start with the simple sine function $y = 1.0\sin(1.0x + 0.00) + 0.0$ for each of these exercises.



a. Click on the start point (an orange ✚) at the origin and drag it upward to the point $(0, 1)$, then downward to the point $(0, -1)$, and then back to the origin. What changes does this cause to the equation and the graph?

b. Click on the start point at the origin and drag it toward the left to the point $(-\pi, 0)$, then toward the right to the point $(\pi, 0)$, and then back to the origin. What changes does this cause to the equation and the graph?

c. Click on a point on the graph and drag it upward; then drag it downward. What changes does this cause to the equation and the graph?

d. Click on a point on the graph and drag it to the left; then drag it to the right. What changes does this cause to the equation and the graph?

2. Start with the simple sine function, $y = 1.0 \sin(1.0x + 0.00) + 0.0$, for each of these exercises.

   a. Change the value of A from 1.0 to 1.5, then to 2.0, then to 0.6, then to 0.3, and then back to 1.0. How does changing A affect the graph?

   b. Change the value of B from 1.0 to 1.5, then to 2.0, then to 3.0, then to 0.5, and then back to 1.0. How does changing B affect the graph?

   c. Change the value of C from 0.0 to 1.57, then to 3.14, then to 1.5π, then to 6.28, then to -1.57, then to -3.14, and then back to 0.0. How does changing C affect the graph? **Note:** The wheel only moves in increments of 0.1. However, you can enter the given values in the box above the wheel.

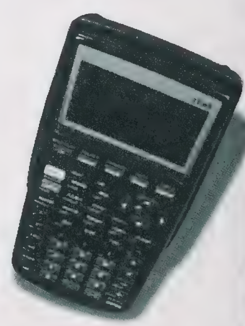
   d. Change the value of D from 0.0 to 0.5, then to 1.0, then to -0.5, then to -1.0, and then back to 0.0. How does changing D affect the graph?

   e. **i.** Which coefficient affects the amplitude of the graph?
      ii. Which coefficient affects the period of the graph?
      iii. Which coefficients affect the starting point of the graph?
      iv. Which coefficient affects the vertical displacement of the graph?

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 94–96.

In junior high you were introduced to scientific notation. The values found by using the Sinusoidal Regression feature on the TI-83 often use scientific notation. This occurs because the regression calculations require small numbers to get the best fit. You will see numbers on your calculator like 2E⁻13 and 1.415783E⁻14. In normal scientific notation, these numbers are written as 2.0×10^{-13} and 1.415783×10^{-14} respectively. In expanded form, these numbers are written as follows:

0.000 000 000 000 2 and 0.000 000 000 000 014 157 83



For the purposes of writing and using sinusoidal regression equations, you will use numbers rounded to 2 or 3 decimal places. That means small values, like those just mentioned, will be considered to be 0.

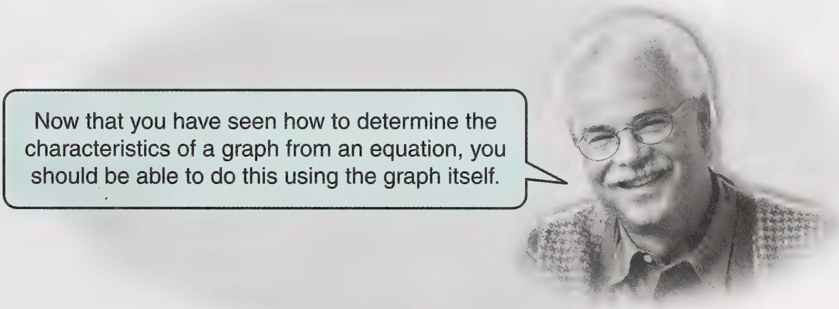
3. Turn to pages 233 to 236 of the textbook and complete exercises 1 to 15 of "Investigation: Exploring the Effect of the Parameters *a*, *b*, *c*, and *d*." **Note:** In exercise 14, the height listed for time 0 is incorrect. It should be 1.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 96–105.

Turn to pages 236 and 237 of the textbook and read the information following the investigation; then work through "Example 1: Determine the Characteristics From an Equation."

Remember to turn all plots off when doing part b. of Example 1 on your calculator.

Turn to page 6 of Assignment Booklet 5A and answer question 12.



Turn to page 238 of the textbook and work through "Example 2: Determine the Characteristics From a Graph." Here you will see how to use a graph to determine the characteristics of a sinusoidal curve.

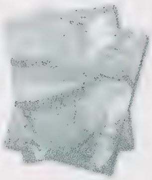
Turn to page 6 of Assignment Booklet 5A and answer question 13.

4. Turn to pages 238 to 240 of the textbook and answer the following.

   a. exercises 1 and 2 of "Discussing the Ideas"

   b. exercises 1.a., 1.b., 1.d., 2.a., 2.d., 3, 5, and 6 of "Exercises: Checking Your Skills"

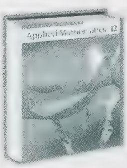
Compare your responses with the suggested answers in the Appendix, Activity 4, pages 105–108.



Turn to pages 7 to 10 of Assignment Booklet 5A and answer question 14.

Looking Back

In this activity you have studied the characteristics of sinusoidal graphs and how they relate to the equation $y = a\sin(bx + c) + d$.



5. Turn to page 240 of the textbook and answer "Communicating the Ideas."

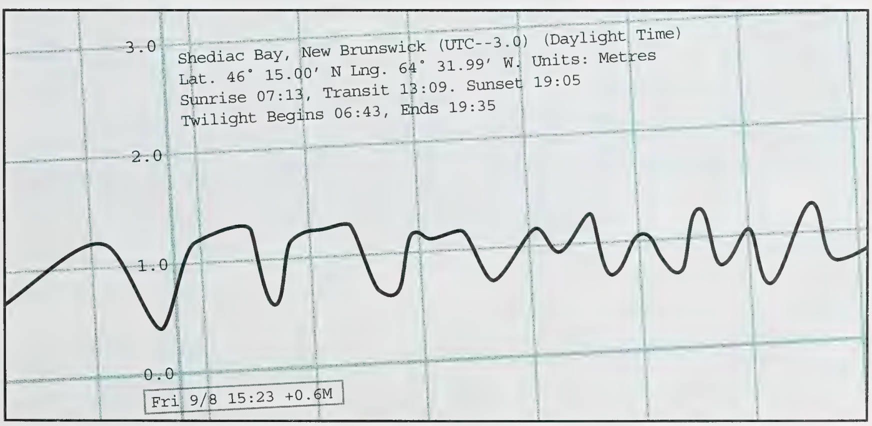
Compare your response with the suggested answer in the Appendix, Activity 4, page 108.

ACTIVITY 5

Applications Involving Sinusoidal Data

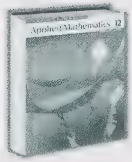
The ocean waves you see and hear when you visit one of Canada's three coasts can be very relaxing or extremely scary. It depends on whether you are watching them or trying to keep your boat from succumbing to them. The depth of water in harbours is of tremendous importance to shipowners. They study tide tables and depth charts to be sure that their ships will make it safely in and out of the ports they travel to.

Scientists who study tides create computer-generated, sinusoidal-type graphs of the tide level versus time.



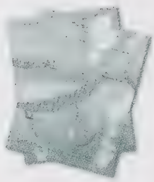
There are many times that using a gentle curving periodic function will allow you to make predictions that are extremely useful. In fact, many computer programs that predic tides use multiple sine waves in calculating their results.

In this activity you will analyse problems that involve sinusoidal data.



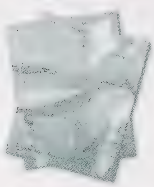
Turn to page 241 of the textbook and read the introductory paragraph of Tutorial 5.5, "Applications." Then work through "Example 1: AC Voltage." (You will probably find the amplitude to be 145 V, not 160 V.)

In Activity 4, you were shown how to determine a sine equation from a sinusoidal graph Simply look for the important features of the graph, and you can write the equation.



Turn to page 1 of Assignment Booklet 5B and answer questions 1 and 2.

Turn to page 242 of the textbook and work through "Example 2: Average Daily Temperatures at Eureka."



Turn to page 1 of Assignment Booklet 5B and answer question 3.

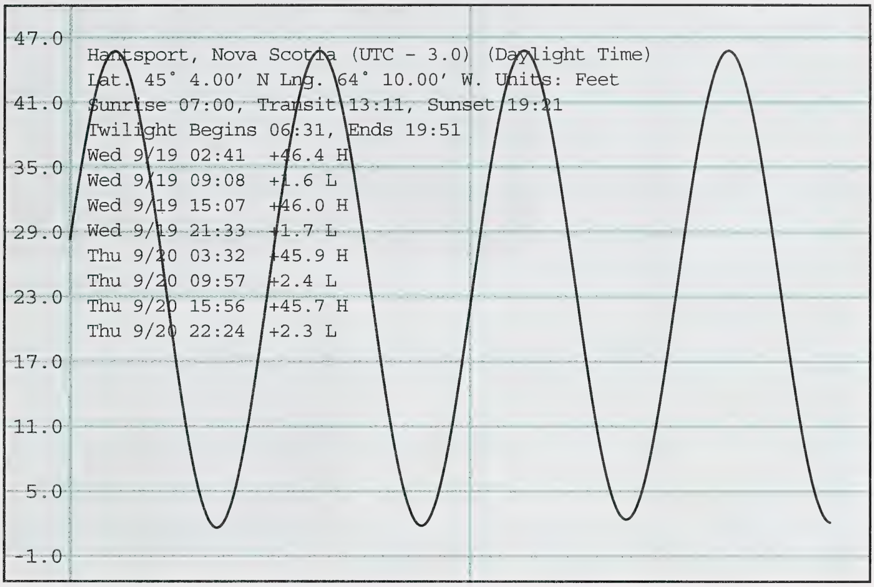
1. Turn to pages 244 to 246 of the textbook and answer the following.

   a. exercise 1 of "Discussing the Ideas"
   b. exercises 2 and 5 of "Exercises: Checking Your Skills"

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 109–111.

Example

The following computer-generated graph shows the water depth at Hantsport, Nova Scotia, for a couple of days in September. What equation would fit this data?



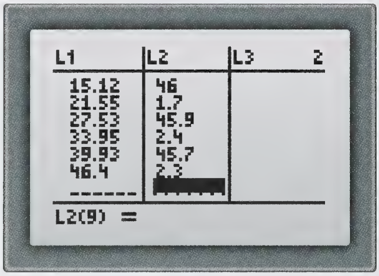
Solution

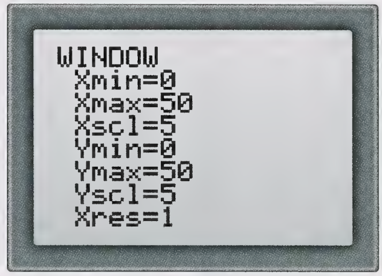
Use the eight high and low points given on the graph to find a sinusoidal regression equation.

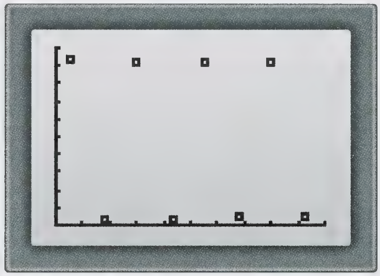
First, convert the times into decimal hours. Remember to add 24 to the times for Thursday.

Time of Day	02:41	09:08	15:07	21:33	03:32	09:57	15:56	22:24
Time (h)	2.68	9.13	15.12	21.55	27.53	33.95	39.93	46.40
Height (feet)	46.4	1.6	46.0	1.7	45.9	2.4	45.7	2.3

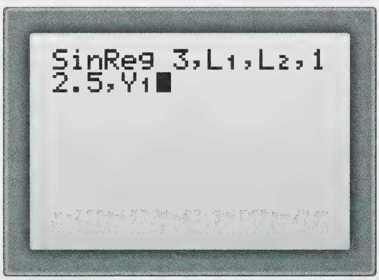
Enter the data into lists L1 and L2, and graph the points.

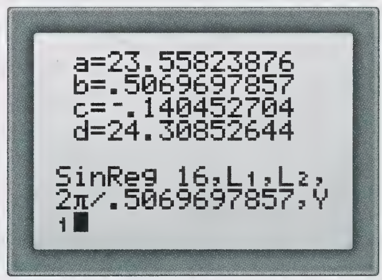


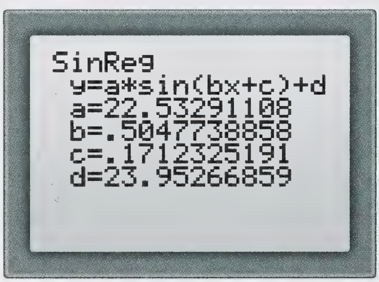


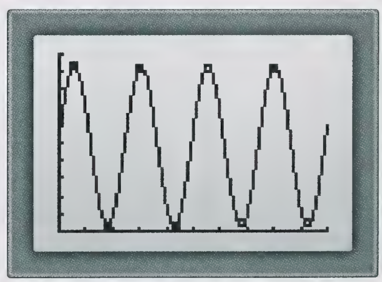


After estimating the period as 12.5 h, apply the SinReg function.



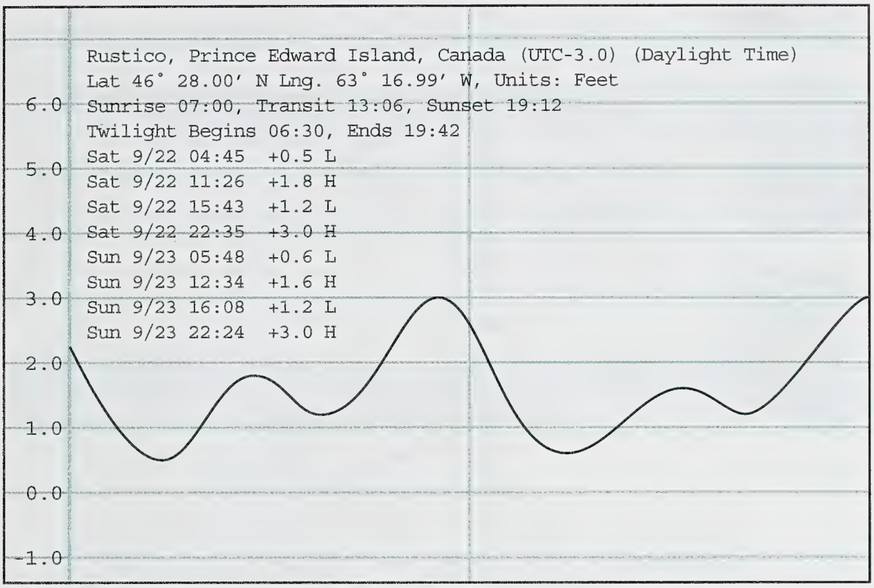






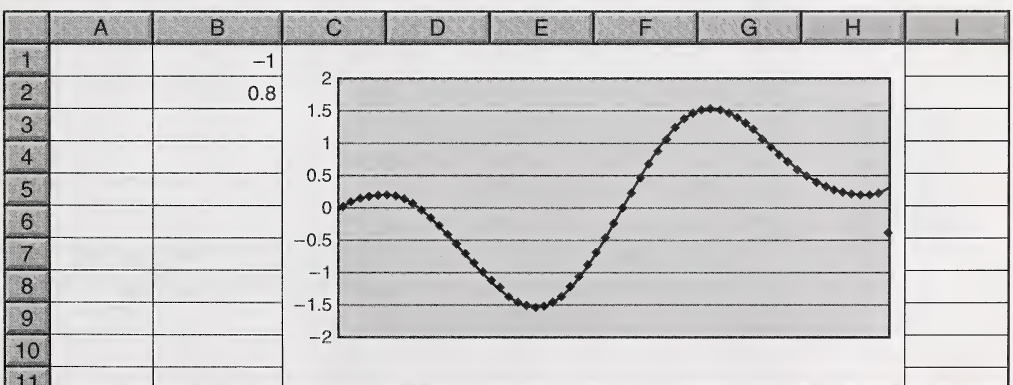
The equation for the tides on these days is $y \doteq 22.53 \sin\left(0.50x + 0.17\right) + 23.95$.

Tides are not always as clearly sinusoidal. Sometimes they have a much more complex pattern. A few days after the data in the preceding example was obtained, the water levels at Rustico on Prince Edward Island were as follows.



These water levels look like two sine waves of different frequencies superimposed on each other. Also, the change in depth of water is nowhere near as much as at Hantsport, Nova Scotia.

If possible, try the spreadsheet *FOURIER* from the Applied Mathematics 30 CD and experiment with creating combinations of sine waves to see if you can approximate the shape of the graph of Rustico, Prince Edward Island. To start, try entering -1 into cell B1 and 0.8 into cell B2; then click on "Make a new curve."



Turn to page 242 of the textbook and read the information at the bottom of the page. Then work through "Example 3: The Apparent Annual Motion of the Sun" on page 243.

Turn to page 2 of Assignment Booklet 5B and answer question 4.

2. Turn to pages 244 to 247 of the textbook and answer the following.

   a. exercises 2 and 3 of "Discussing the Ideas"

   b. exercises 1, 3, and 6 of "Exercises: Checking Your Skills"
   Note: For exercise 1, vertical translation is the vertical distance of the start point from zero and the horizontal translation is the horizontal distance of the start point from zero. For exercise 3.c., use $-15.5°\text{C}$ as the actual average temperature for Winnipeg in January 1999.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 111–116.

Turn to pages 2 to 6 of Assignment Booklet 5B and answer questions 5 to 10.

Looking Back

In this activity you have applied your knowledge of the characteristics of sinusoidal functions to solve real-world problems.

3. Turn to page 240 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 5, page 117.

Module Review

This module dealt with Chapter 5: Sinusoidal Data in the *Addison-Wesley Applied Mathematics 12 Source Book.*

Turn to page 250 of the textbook and review the skills and concepts that were developed in this module. Also, read the important results and formulas that you discovered.

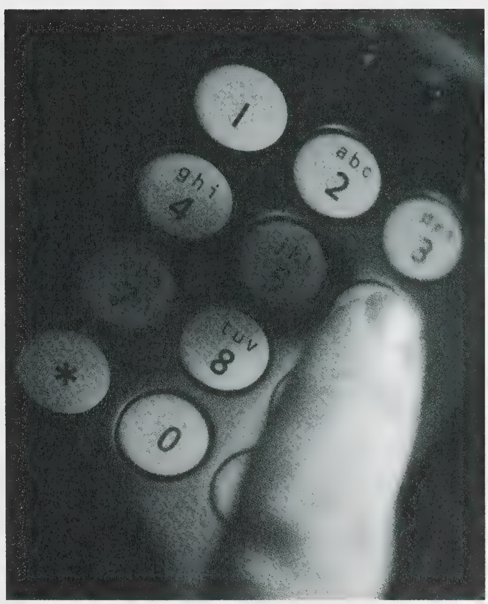
1. Answer exercise 1 of Part A of "What Should I Be Able to Do?" on page 251 of the textbook. **Note:** The time following 12:30 in the table should read 13:00, not 12:00.

2. Answer exercises 2, 4, and 6 of Part B of "What Should I Be Able to Do?" on pages 252 to 254 of the textbook.

Compare your responses with the suggested answers in the Appendix, Module Review, pages 117–123.

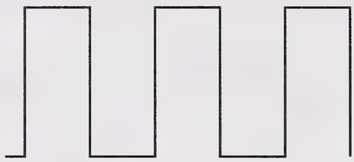
Turn to pages 7 to 18 of Assignment Booklet 5B and complete the Module Review Assignment.

If you had difficulties understanding the skills and concepts in Module 5: Sinusoidal Data, it is recommended that you contact your teacher for some extra help activities. If you have a clear understanding of the skills and concepts in this module, you may wish to do the following enrichment activity. You may decide to do both.



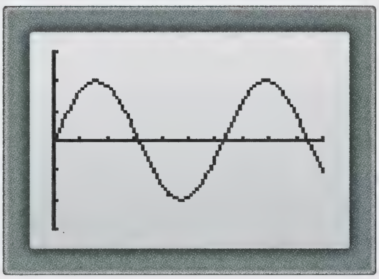
Enrichment

A French mathematician by the name of Jean Baptiste Joseph Fourier (1768–1830) discovered a remarkable thing. He discovered that any smooth, continuous curve can be expressed as a sum of sinusoidal curves. It seems quite counterintuitive to say that a curve like the following is really a sum of sine curves, but it is.

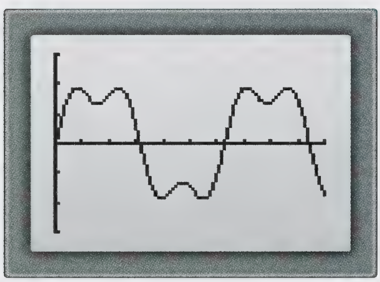


You can see how this builds up by looking at the following series of graphs.

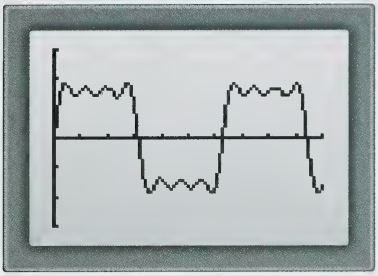
1 Term: $y = \sin x$



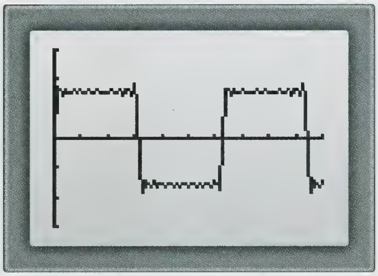
2 Terms: $y = \sin x + \frac{1}{3}\sin(3x)$



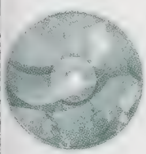
4 Terms: $y = \sin x + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x)$



11 Terms: $y = \sin x + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \ldots + \frac{1}{21}\sin(21x)$



As you can see, the graph looks more and more like the original square wave as more terms are added.



There is a spreadsheet called *FOURIER* on the Applied Mathematics 30 CD that will allow you to experiment with Fourier's synthesis analysis.

What would the shape of a sum of sine waves like the following be? **Hint:** Simply enter the coefficients in column B.

$$\sin x + \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) + \ldots + \frac{1}{31}\sin(31x) + \ldots$$

Compare your responses with the suggested answers in the Appendix, Module Review: Enrichment, pages 123–124.

Completing the Project

By now you should have completed the initial research for the module project, Angle of Elevation of the Sun. You should also have devised a means of holding a metre-stick vertical throughout a day while you take measurements of the length of the shadow the metre-stick casts. For this project, you will be asked to complete three parts. First, you will have to collect data on the lengths of shadows that a metre-stick casts at different times throughout a day. Second, you will have to analyse the data you gathered and find an equation that best describes the data. The final part is to use your findings to make predictions about sun events like sunrise and sunset.

Turn to pages 220 and 221 of the textbook and read "Angle of Elevation of the Sun." Pay particular attention to the information regarding gathering data involving the sun. Then complete the observations as detailed in exercises 3 to 5 and store the results in the project section of your mathematics binder.

You can now continue to answer exercises 6 to 10 on page 221. Again, store the results i the project section of your mathematics binder.

You will require your responses to exercises 1 to 10 from pages 220 and 221 of the textbook to answer questions in the Module Project section of Assignment Booklet 5B.

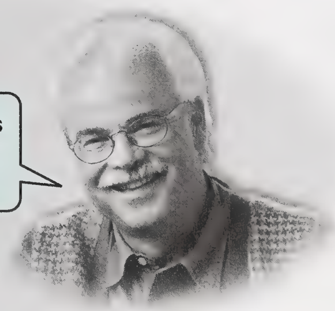
Turn to pages 255 and 256 of the textbook and answer exercises 7, 8, and 9 of Part C of "What Should I Be Able to Do?" **Note:** The table in Example 3 is actually on page 243, not page 277.

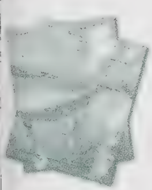
Compare your response with the suggested answer in the Appendix, Module Project, pages 124–125.

Module Project

Now that you have worked through some exercises to help you gain insight into the module project, you may wish to revise information you have developed and stored in the project section of your mathematics binder. When you have finished the revisions, complete the module project, Angle of Elevation of the Sun.

Use your responses from the textbook exercises on pages 208, 220, 221, 255, and 256 to assist you in the completion of the module project.



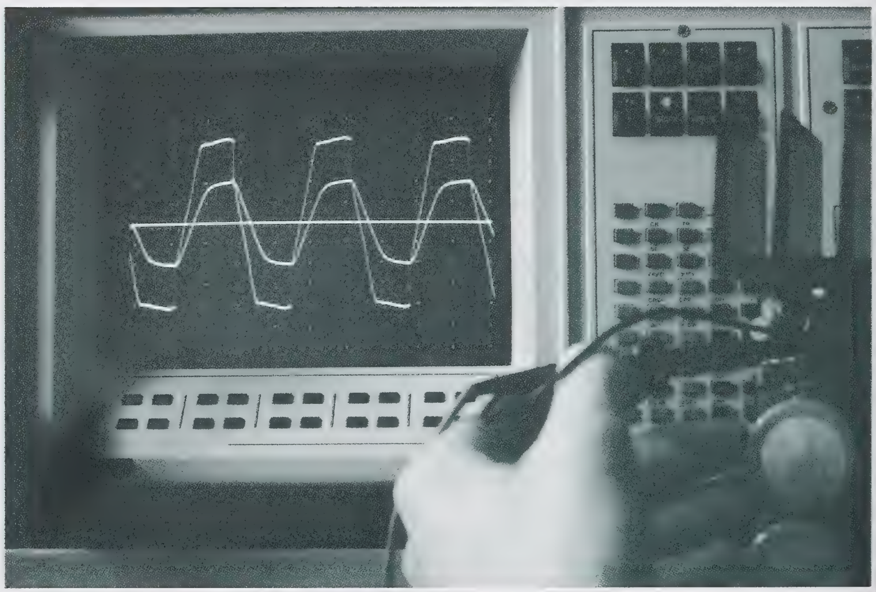


Turn to pages 19 to 23 of Assignment Booklet 5B and complete the module project.

MODULE SUMMARY

In this module you studied periodic and, in particular, sinusoidal events. You graphed data of periodic events and analysed them. You found the period of repetitive data and estimated values for parts of events that were not in the recorded data. You examined radian measure and the meaning of amplitude and period as they relate to sinusoidal curves. You also examined the characteristics of sinusoidal graphs and how they relate to the equation $y = a \sin(bx + c) + d$. You have learned how to fit sinusoidal regression equations to periodic data and how these equations can be used to help solve problems.

Understanding the characteristics of sinusoidal graphs and how they relate to the equation $y = a \sin(bx + c) + d$ is a requirement for various professionals, such as economists, engineers, and mathematicians. These professionals can use their knowledge to make predictions about periodic data related to their field of study.



APPENDIX

Glossary
Suggested Answers
Image Credits

Glossary

amplitude: half the difference between the maximum and minimum values of a periodic function

cycle: one period

cyclic: repeating at regular intervals

maximum value: the largest value of a function or set of data

median value: the average of the maximum and minimum values of a periodic function

minimum value: the smallest value of a function or set of data

period: the smallest interval over which a periodic function repeats its values

periodic: repeating at regular intervals

periodic data: data that repeats its values over a particular interval

periodic event: an event that repeats its values over a particular interval

radian measure: a system for measuring angles using the ratio of the arc length to the radius

sampled data: data that is only a part of the available information

sine curve: a graph from the family of functions of the form $y = a\sin(bx + c) + d$

sinusoidal event: an event that produces a sine curve when graphed

sinusoidal regression: a method of finding a best-fit sine function for a given set of data

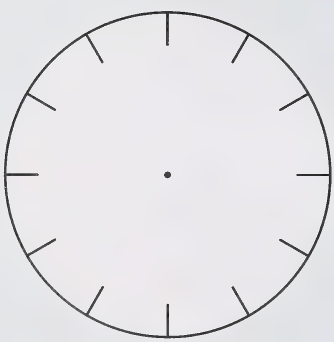
Suggested Answers

Activity 1: Collecting and Plotting Periodic Data

1. Textbook exercises 1 to 6 of "Investigation 1: Patterns in a Machine: Clock Time," pp. 210 and 211

The diagrams are drawn smaller to conserve space.

1.

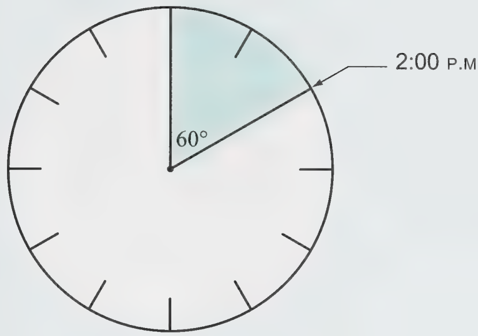


2. $360° \div 12 = 30°$

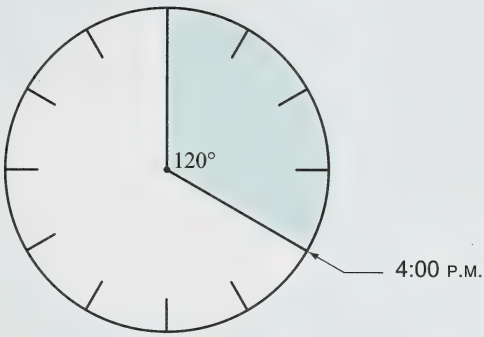
The hour hand passes through 30° in 1 h.

3. **a.**

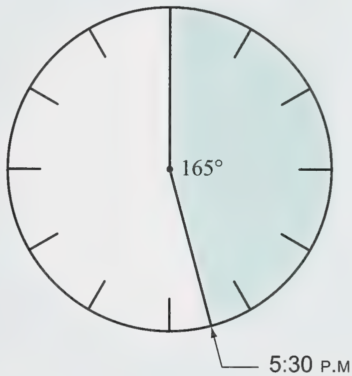
b.



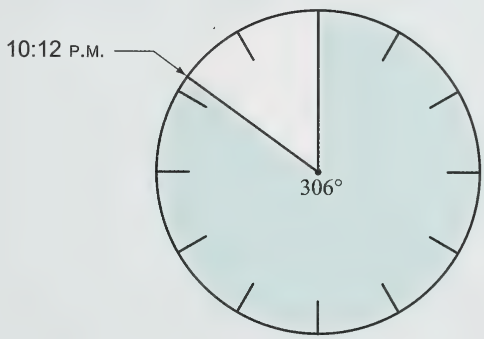
c.



d.



e.



$$30° \times \frac{12}{60} = 30° \times 0.2$$
$$= 6°$$

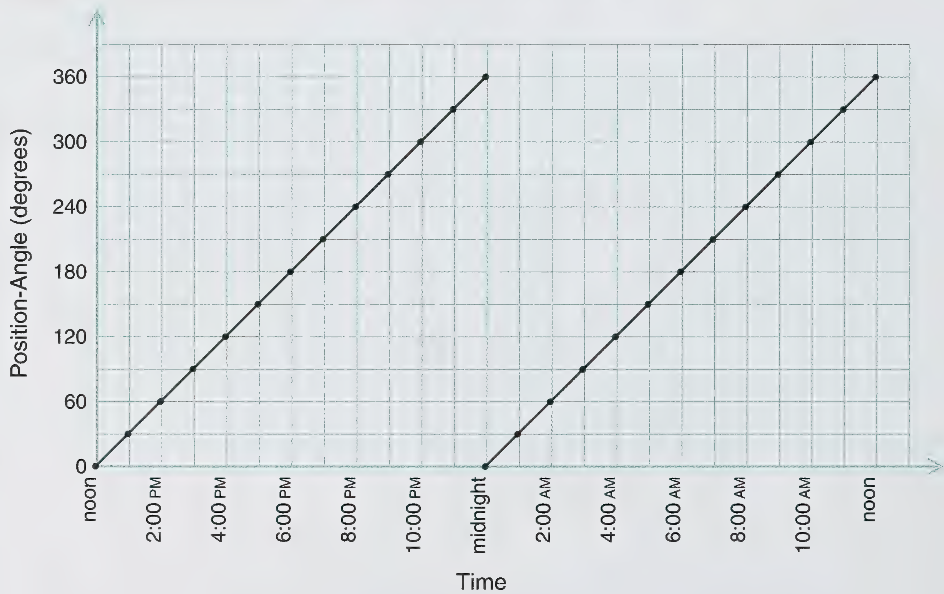
$$300° + 6° = 306°$$

Activity 1 (continued)

4.

Year	Position Angle
noon	0°
1:00 P.M.	30°
2:00 P.M.	60°
3:00 P.M.	90°
4:00 P.M.	120°
5:00 P.M.	150°
6:00 P.M.	180°
7:00 P.M.	210°
8:00 P.M.	240°
9:00 P.M.	270°
10:00 P.M.	300°
11:00 P.M.	330°
midnight	360°, 0°
1:00 A.M.	30°
2:00 A.M.	60°
3:00 A.M.	90°
4:00 A.M.	120°
5:00 A.M.	150°
6:00 A.M.	180°
7:00 A.M.	210°
8:00 A.M.	240°
9:00 A.M.	270°
10:00 A.M.	300°
11:00 A.M.	330°
noon	0°

5.

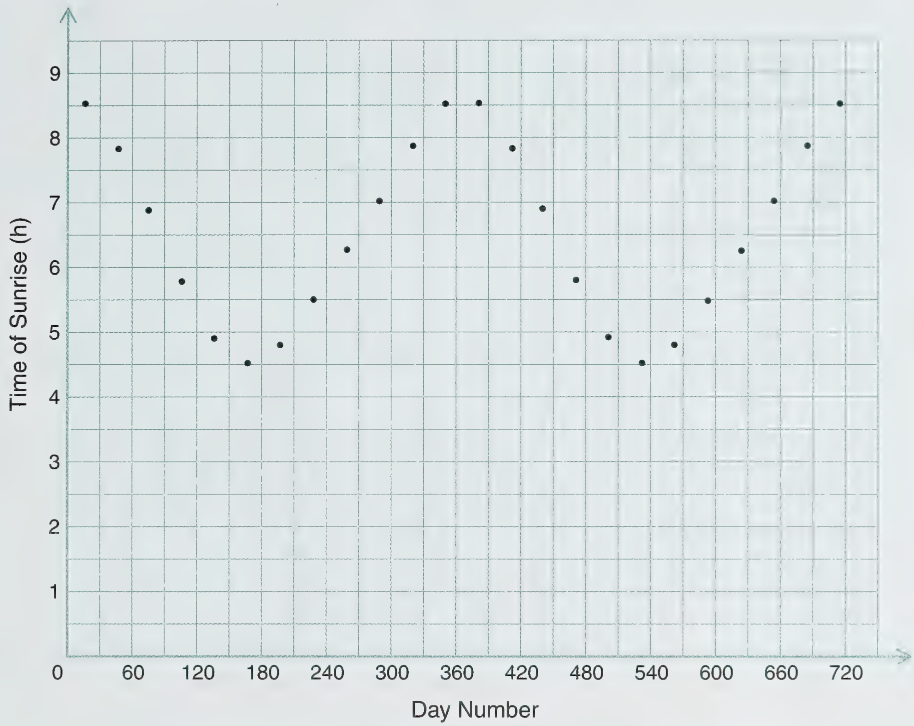


Yes, the points should be connected because the hour hand moves through intermediate points.

6. The graph starts at 0° and rises linearly to 360°. It repeats this pattern a second time for the next twelve-hour period.

2. **Textbook exercises 1 to 6 of "Investigation 2: Patterns in Nature: Times of Sunrise," pp. 211 and 212**

1.



2. No, the points should not be connected because the sunrise for a given day occurs only once.

3. This graph is wave-like and repeats every year.

4. March 15, 2000, would be day number 805 (16 days from December 15, 1999, to January 1, 2000, and 75 days from January 1, 2000, to March 15, 2000). **Remember:** The year 2000 was a leap year. According to the graph, day 75 is approximately 6.8 h, which corresponds to 6:48 A.M. $(0.8 \times 60 = 48)$.

5. The table suggests that the time would be about 6.90 h, which corresponds to 6:54 A.M. (6.8 h and 6.90 h are within a few minutes of each other.)

6. On March 15, 2000, sunrise in Brandon, Manitoba, occurred at 6:52 A.M.

3. Textbook exercises 1 and 4 of "Exercises: Checking Your Skills," pp. 216 to 218

1. a. This is a periodic function. There are 20 spaces for 0.005 s.

$$\frac{0.005}{20} = 0.000\ 25 \text{ s/space}$$

There are 11 spaces from the start to the end of the pattern.

$$0.000\ 25 \times 11 = 0.002\ 75$$

It has a period of about 0.002 75 s.

b. This is not a periodic function. The vertical motion does not repeat, the height decreases and looks as though it will end.

c. This is considered a periodic function. It has a period of about 32 years. Although the pattern does not repeat exactly, it will be useful to consider this function to be periodic because it will provide some idea as to when the number of sunspots will peak. The pattern extends from the highest peak to the next highest peak.

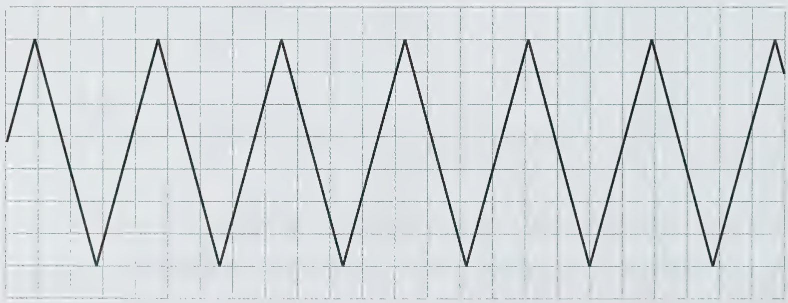
4. a. The graph shows that inhaling and exhaling takes 5 s.

b. Because the breathing rate is fairly slow and the change in volume is moderate, you would conclude that the person is resting, or is involved in a gentle activity.

4. Textbook exercises 1, 2, and 3 of "Discussing the Ideas," p. 216

1. Patterns in sets of data allow you to predict events that may occur or might have occurred at times that are not represented by the data.

2. Not all periodic data are sinusoidal. Here is an example of a non-sinusoidal periodic function.



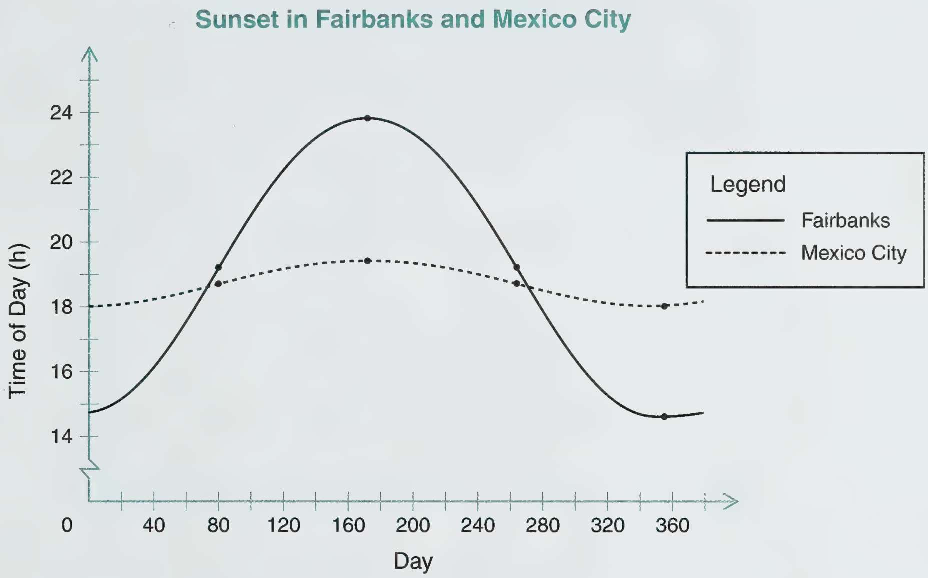
3. All sinusoidal data is periodic.

5. a. **Textbook exercises 3, 5, and 7 of "Exercises: Checking Your Skills," pp. 217 to 219**

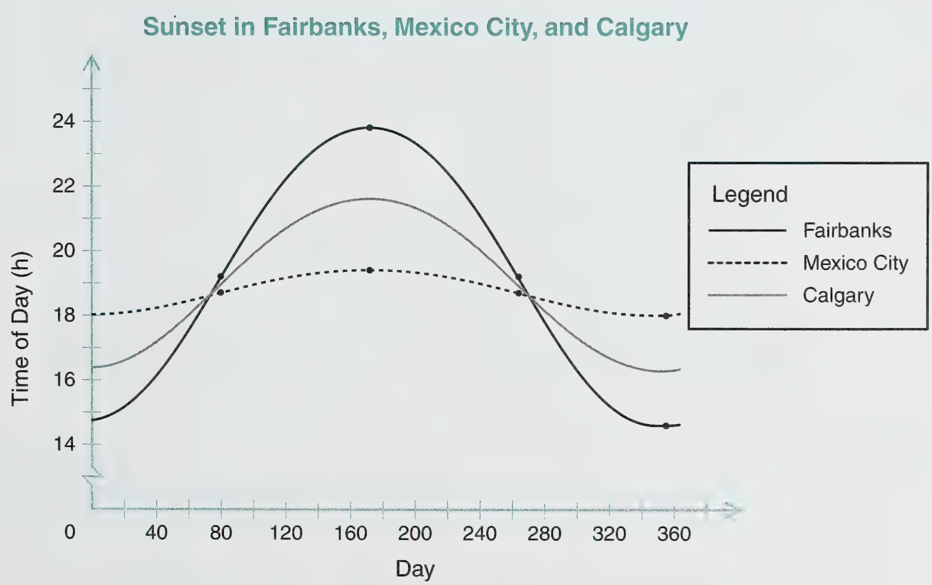
3. a. When a person exercises, the heart rate increases as well as the blood pressure. This would make the graph increase in height and the period decrease.

   b. The new graph would have a similar shape to the current graph, and it would still be periodic.

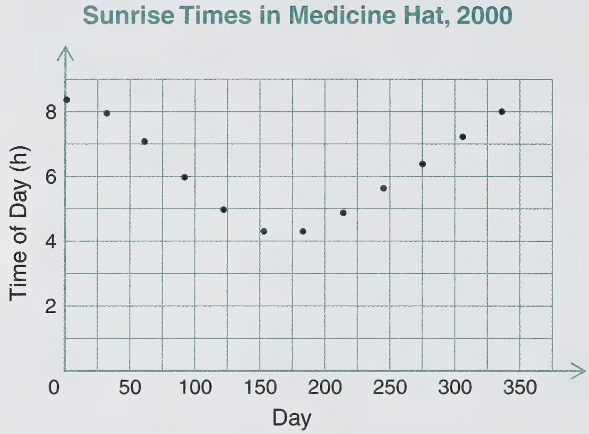
5. a.



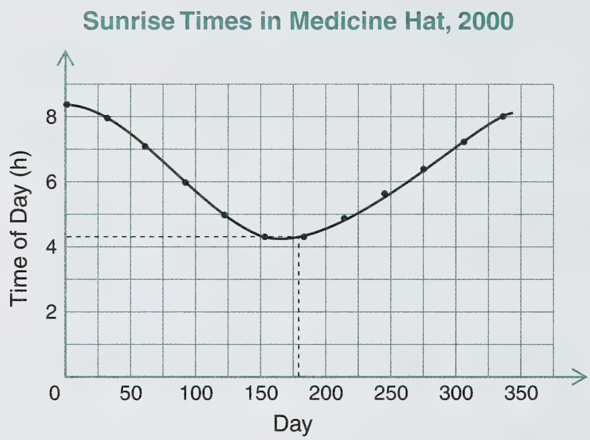
   b. Because Calgary lies about halfway between Fairbanks and Mexico City, the graph of the sunsets in Calgary should also lie about halfway between the graphs of Fairbanks and Mexico City.



7. a.



b. June 27 is the $153 + 26 = 179\text{th}$ day of the year.



Therefore, the sunrise time, according to the graph, is about 4:15 A.M. (4.25 h).

c. The sunrise on January 1, 2001, would be expected to be slightly earlier than the sunrise on January 1, 2000, since 2000 was a leap year. Therefore, a good estimate would be for Tela to expect the sunrise to occur at about 8:18 A.M. (8.30 h).

b. Textbook exercise 8 of "Exercises: Extending Your Thinking," p. 219

8. a. You would expect to find a periodic graph. The graph would probably be sinusoidal.

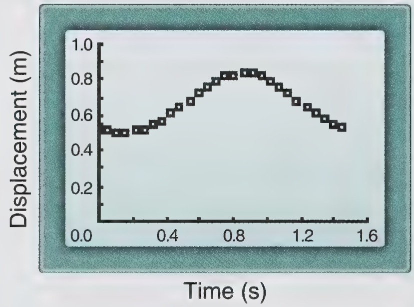
b. The pendulum swinging from one side to the other and back to the starting point would correspond to one cycle of the graph.

c. The pendulum's highest points on both sides of its swing would correspond to the maximum horizontal displacement.

d. Answers will vary. The following pendulum data was collected using a CBR.

Time (s)	Displacement (m)
0	0.526 94
0.053 76	0.510 19
0.107 52	0.504 15
0.161 28	0.503 18
0.215 04	0.509 91
0.268 80	0.521 86
0.322 56	0.543 82
0.376 32	0.570 60
0.430 08	0.605 06
0.483 84	0.642 95
0.537 60	0.683 58
0.591 36	0.721 48
0.645 12	0.758 55
0.698 88	0.790 41
0.752 64	0.814 98
0.806 40	0.828 30
0.860 16	0.836 54
0.913 92	0.831 59
0.967 68	0.818 83
1.0214	0.791 78
1.0752	0.758
1.129 10	0.724 37
1.1827	0.685 37
1.2365	0.646 93
1.2902	0.609 59
1.3440	0.575 13
1.3978	0.546 02
1.4515	0.525 70

The following shows a time-displacement graph of this data. As you can see, the shape is sinusoidal.



6. Textbook exercise "Communicating the Ideas," p. 219

Answers will vary.

Periodic data has a pattern that repeats over and over again. The following diagram shows a periodic graph with almost two complete cycles.

This is often more easily seen in a graph than in raw data. Sinusoidal data is periodic data with a very smooth, wave-like shape. The following graph shows two cycles of a sine graph.

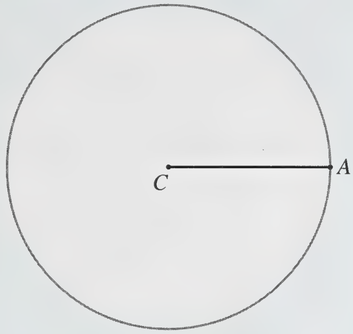
To find the period of periodic data, you generally work with the graph of the data. Look for an outstanding feature in one cycle of the graph, and measure the distance from this feature to the same feature in the next cycle of the graph.

Activity 2: Radian Measure and Sine Curves

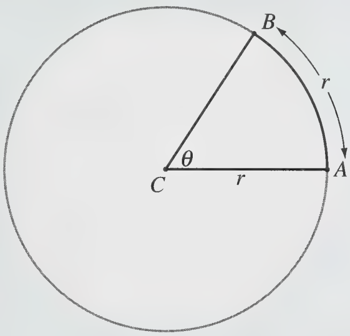
1. Textbook exercises 1 to 10 of "Investigation 1: What Is a Radian?," pp. 222 and 223

Diagrams may vary slightly. Sample diagrams given here are half the size requested.

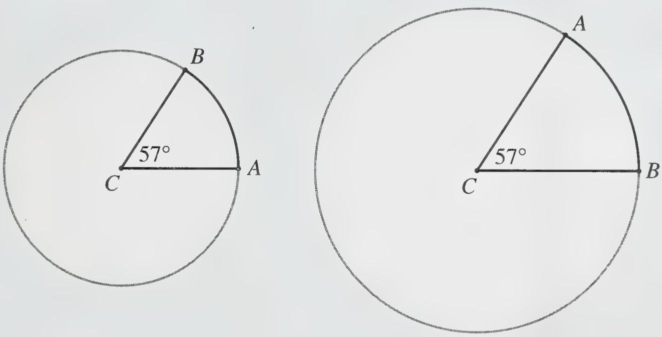
1. to 3.



4.



5.



The angle measure is 57° in each circle.

6. The size of the circle did not affect the results. The ease of matching the string length and the arc on a circle will change with the size of the circle, however.

7. There are about 57° in a radian (57.295 779 51° to be more accurate).

8. The standard formula for the circumference of a circle is $C = 2\pi r$, so the circumference is $2\pi r$.

9. The circumference will hold 2π arcs of length r.

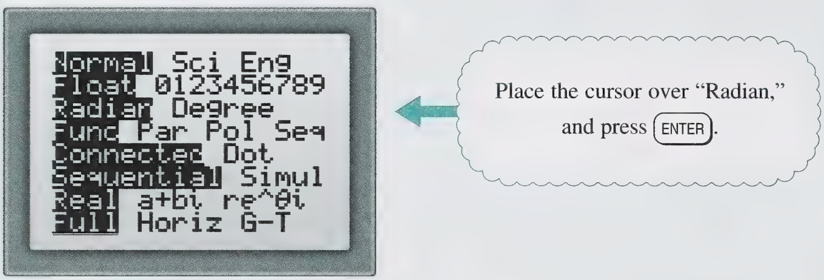
10. 2π radians will fit into one complete revolution.

2. The animation shows a grooved wooden spool with a string fastened to it. The spool is rotated through an angle so that exactly one radius worth of string is wound around the spool. The arc is shown to be the same length as the amount of string wound up.

   This animation shows that an angle that measures one radian is created by an arc that has a length of one radius.

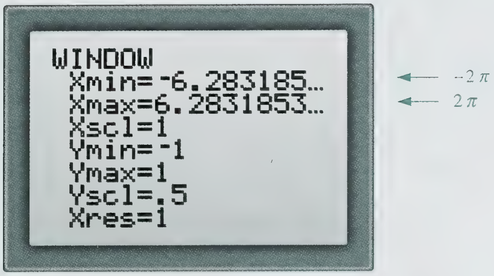
3. **Textbook exercises 1 to 8 of "Investigation 2: The Characteristics of $y = \sin x$," pp. 223 and 224**

   1.

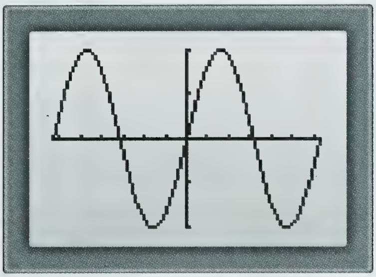


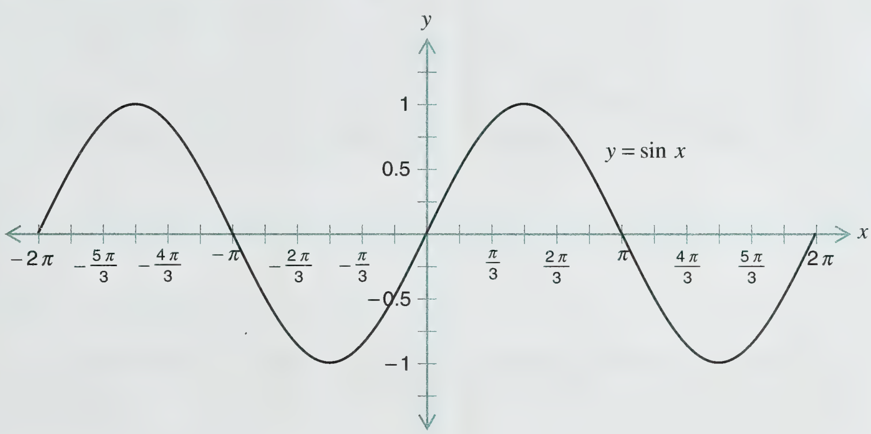
   2.

3.


WINDOW
Xmin=-6.283185…
Xmax=6.2831853…
Xscl=1
Ymin=-1
Ymax=1
Yscl=.5
Xres=1
−2π
2π

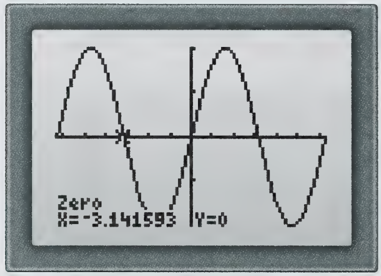
4.

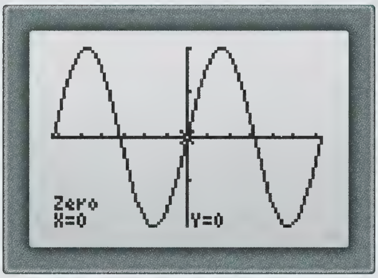


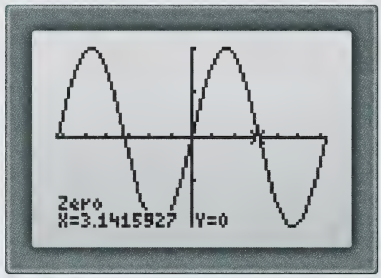

y
x
1
0.5
−0.5
−1
y = sin x
−2π
−5π/3
−4π/3
−π
−2π/3
−π/3
π/3
2π/3
π
4π/3
5π/3
2π

5. The x-intercepts are $-2\pi, -\pi, 0, \pi,$ and 2π. **Note:** Because the boundaries used for graphing exactly match -2π and 2π, the Zero feature of the TI-83 will not allow you to check that they are also zeros unless you adjust the window settings.

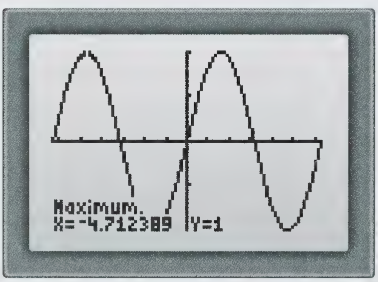
Remember: Press 2nd [CALC] 2 to obtain the Zero feature.

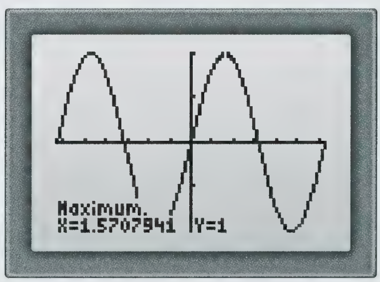


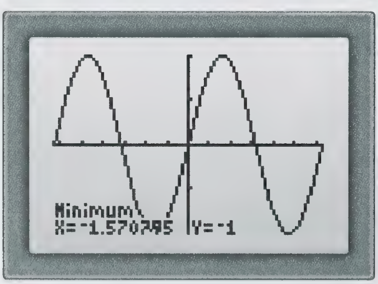


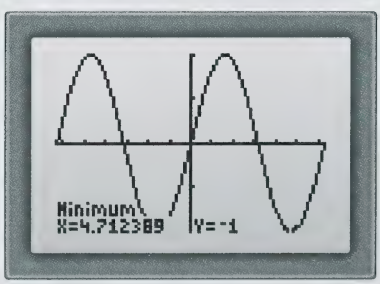


6. The maximum values of the sine function are at $\left(-\frac{3\pi}{2}, 1\right)$ and $\left(\frac{\pi}{2}, 1\right)$. The minumum values for the sine function are at $\left(-\frac{\pi}{2}, -1\right)$ and $\left(\frac{3\pi}{2}, -1\right)$.









7. $\text{Amplitude} = \dfrac{\text{Maximum value} - \text{Minimum value}}{2}$

$$\begin{aligned} &= \frac{1-(-1)}{2} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

The amplitude of $y = \sin x$ is 1.

8. The period of the function $y = \sin x$ is 2π. This can be seen from finding the distance between two successive maximum values or two successive minimum values.

Method 1: Using Maximum Values

The x-coordinates of two successive maximum points of $y = \sin x$ are $-\frac{3\pi}{2}$ and $\frac{\pi}{2}$.

$$\begin{aligned} \therefore\ \text{Period} &= \frac{\pi}{2} - \left(-\frac{3\pi}{2}\right) \\ &= \frac{\pi}{2} + \frac{3\pi}{2} \\ &= \frac{4\pi}{2} \\ &= 2\pi \end{aligned}$$

Method 2: Using Minimum Values

The x-coordinates of two successive minimum points of $y = \sin x$ are $-\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

$$\begin{aligned} \therefore\ \text{Period} &= \frac{3\pi}{2} - \left(-\frac{\pi}{2}\right) \\ &= \frac{3\pi}{2} + \frac{\pi}{2} \\ &= \frac{4\pi}{2} \\ &= 2\pi \end{aligned}$$

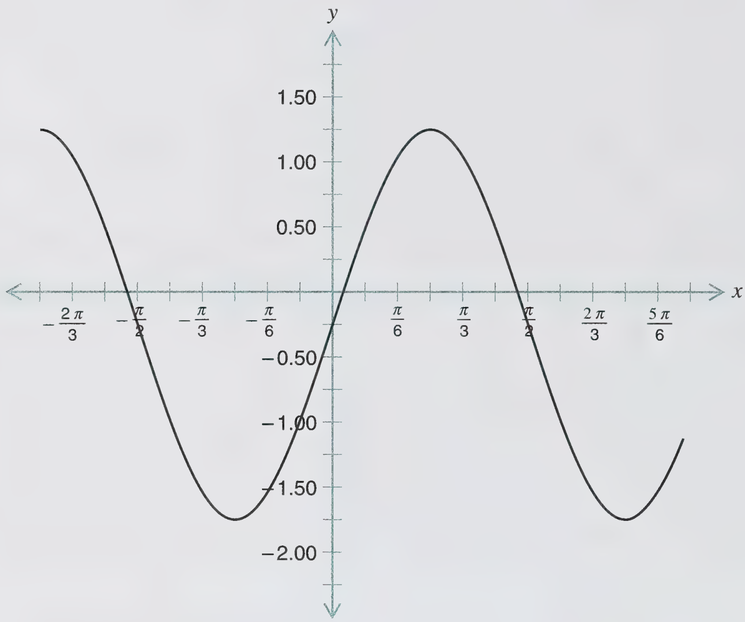
4. Textbook exercises 1 to 4 of "Discussing the Ideas," p. 224

1. A radian is the same size regardless of the size of the circle because the ratio of the circumference to the radius is a constant value, 2π . As the radius increases, so does the circumference.

Activity 2 (continued)

2. Answers will vary. A sample answer is given.

The amplitude and the maximum value do not have to be equal. For example, the following graph has a maximum value of 1.25 and an amplitude of 1.50.



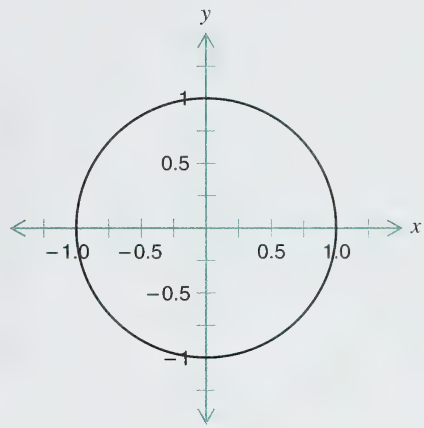
3. The amplitude is always positive because it is defined as half the distance from the maximum value to the minimum value. The maximum will never be smaller than the minimum, so the amplitude will never be negative.

4. The period of a function is the horizontal distance between two identical points in consecutive cycles of the periodic function. The maximum points are identical points in the sine function, so using them to find the period is a good idea.

5. **Textbook exercise "Communicating the Ideas," p. 224**

A radian is a unit of angular measure. It is based on arc length in circles and is used to measure angles.

The circumference of the circle on the right, with a radius of 1.0, is exactly 2π. This means that the radian measure of a complete circle must be 2π, since it is the ratio of the arc length of the angle to the radius of the circle. The degree measure of a complete circle is 360°. This provides a relationship between degrees and radians: 2π radians = 360°.



$$\therefore\ 1 \text{ rad} = \frac{360°}{2\pi} \doteq 57.295\,779\,51° \qquad \text{and} \qquad 1° = \frac{2\pi \text{ rad}}{360} \doteq 0.174\,532\,925 \text{ rad}$$

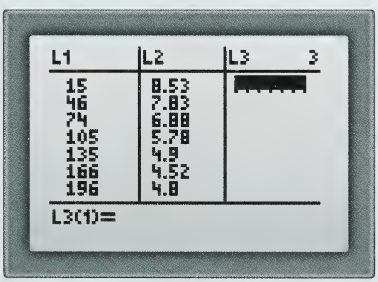
Activity 3: Fitting Sine Curves to Data

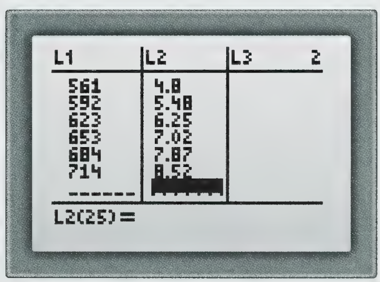
1. **Textbook exercises 1 to 6 of "Investigation 1: Times of Sunrise," pp. 225 and 226**

   1. Press STAT 1 (1:Edit…), and enter the data in the table.

      Before you enter the data, clear list L1 by placing the cursor over "L1" and pressing CLEAR ENTER. Do the same for list L2.

      Now, enter the data.

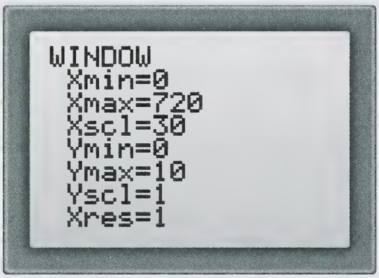


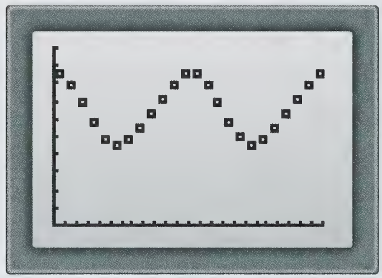


Activity 3 (continued)

Press 2nd [STAT PLOT] ENTER, and choose the following settings for your scatterplot.

Using the window settings given, graph the data. **Note:** Make sure all of the data points are displayed to see a pattern.



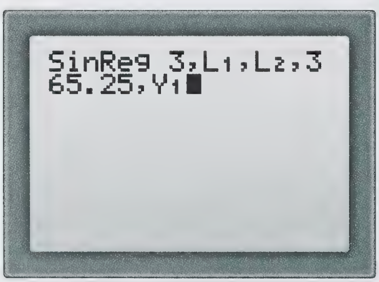


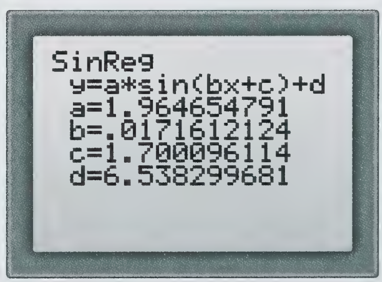
2. SinReg (*number of iterations*, *independent data*, *dependent data*, *period*, *where to store the result*).

Select the CALC menu.

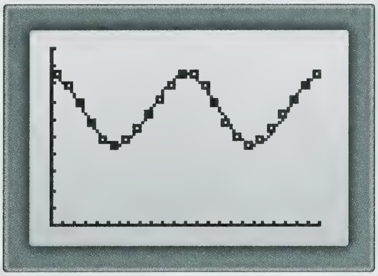
STAT ▶ ALPHA [C] (C:SinReg) 3 , 2nd [L1] , 2nd [L2] , 3 6 5 • 2 5 , VARS ▶ 1 (1:Function) 1 (1: Y_1) ENTER

Select the Y-VARS menu.

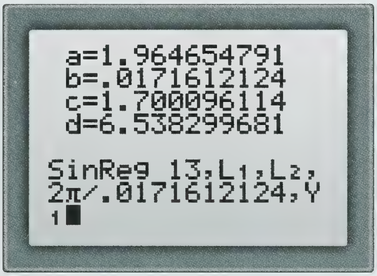


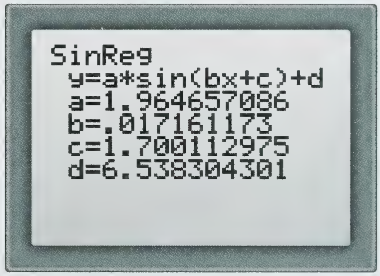


3. Press GRAPH to graph the regression equation you determined in exercise 2. It will take a few seconds to display.



Now, get a better-fit regression equation by changing the parameters given to SinReg (as shown in the following displays).

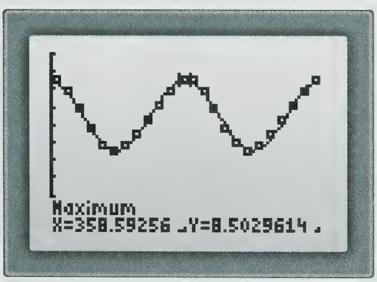




Note: By keeping Y_1 in the parameters, the regression equation will be updated to the better-fit regression equation.

Because the values are only slightly different than the initial regression equation, there is no need to try for any further better-fit regression equations.

Determine the maximum value.

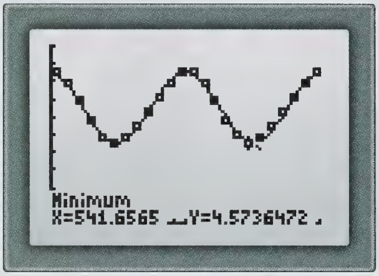


Remember: Press 2nd [CALC] each time you want to determine a minimum or a maximum.

The maximum value occurs at about $(358.59, 8.50)$.

Activity 3 (continued)

Determine the minimum values.



The minimum values occur at about $(175.53, 4.57)$ and $(541.66, 4.57)$.

The period can be calculated from the x-values of the two consecutive minimums.

$$\begin{aligned}\text{Period} &\doteq 541.656\,56 - 175.528\,57\\ &\doteq 366.127\,93\\ &\doteq 366\end{aligned}$$

$$\begin{aligned}\text{Amplitude} &= \frac{\text{Maximum value} - \text{Minimum value}}{2}\\ &\doteq \frac{8.502\,961\,4 - 4.573\,647\,2}{2}\\ &\doteq \frac{3.929\,314\,2}{2}\\ &\doteq 1.964\,657\,1\\ &\doteq 1.96\end{aligned}$$

4. The value of a in the regression equation is about 1.964 657 086 and is a very close match to the amplitude found in exercise 3.

5. $$\begin{aligned}\frac{2\pi}{b} &\doteq \frac{2\pi}{0.017\,161\,173}\\ &\doteq 366.127\,962\,7\end{aligned}$$

The calculated value is a very close match to the period found in exercise 3.

6. Median value $= \dfrac{\text{Maximum value} + \text{Minimum value}}{2}$

$$\doteq \frac{8.502\,961\,4 + 4.573\,647\,2}{2}$$

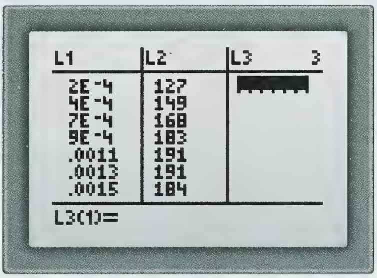
$$\doteq \frac{13.076\,608\,6}{2}$$

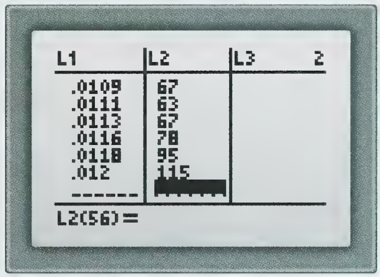
$$\doteq 6.538\,304\,3$$

The median time of sunrise is about 6.54.

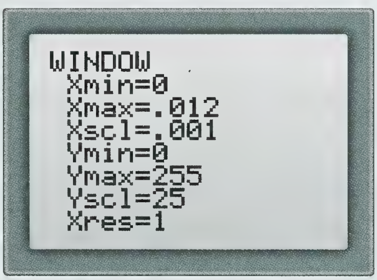
2. Textbook exercises 3, 4, and 5 of "Investigation 2: Determine the Frequency of a Musical Note," p. 226

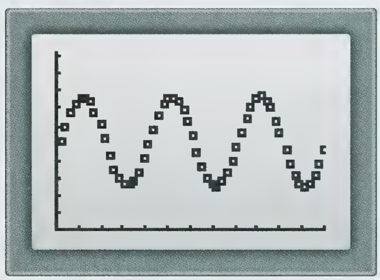
3. Enter the time in list L1 and the CBL values in list L2. Notice that the first few entries are displayed in scientific notation.





Using the window settings given, display the graph.





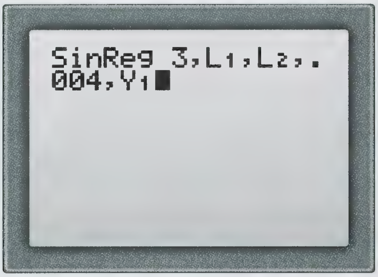
There appear to be three cycles within 0.012 s. Therefore, the period is $0.012 \div 3 = 0.004$ s.

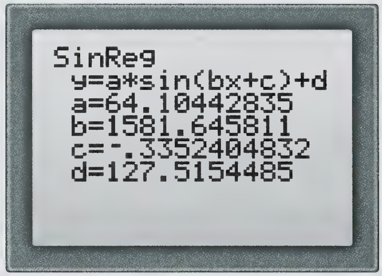
4.

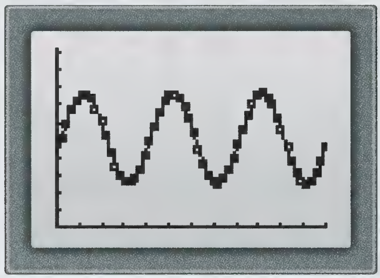
Select the CALC menu.

STAT ▶ ALPHA [C] (C:SinReg) 3 , 2nd [L1] , 2nd [L2] , • 0
0 4 , VARS ▶ 1 (1:Function) 1 (1: Y_1) ENTER

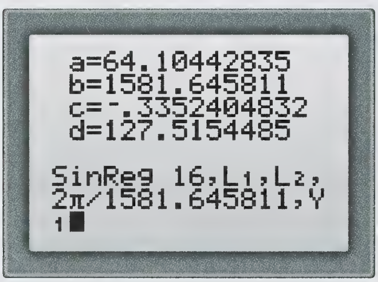
Select the Y-VARS menu.

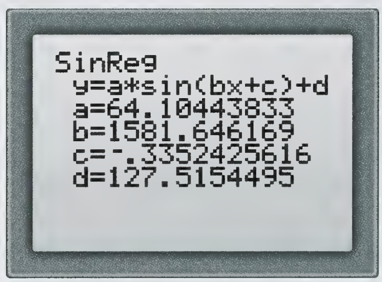






Obtain a better-fit regression equation.

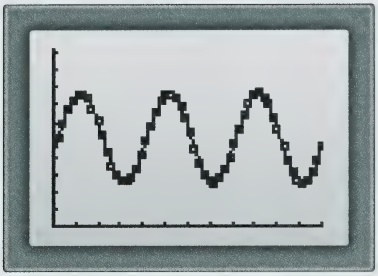




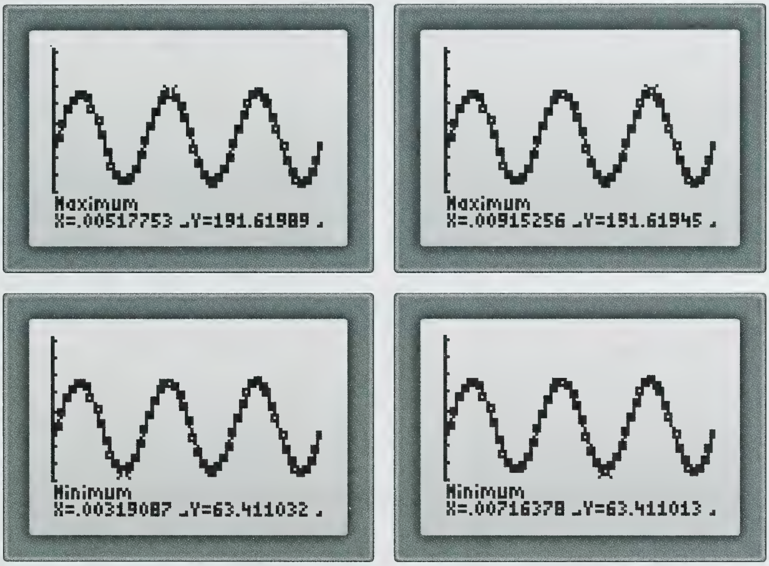
Because the values are only slightly different than the initial regression equation, there is no need to try for any further better-fit equations.

Therefore, the regression equation is $y \doteq 64.10 \sin\left(1582x - 0.3352\right) + 127.5$.

Now graph the regression.



5.



The maximum value is about 192, and the minimum value is about 63.

Notice that the two calculated maximum values are slightly different. This relates to the way the calculator determines the maximum values from what is graphed on the screen. These values will also vary slightly for different calculations for the same point. The first of these values will be used in the following calculations.

$$\begin{aligned}\text{Amplitude} &= \frac{\text{Maximum value} - \text{Minimum value}}{2}\\ &\doteq \frac{191.619\,89 - 63.411\,032}{2}\\ &\doteq 64.104\,429\\ &\doteq 64.1\end{aligned}$$

$$\text{Median value} = \frac{\text{Maximum value} + \text{Minimum value}}{2}$$
$$\doteq \frac{191.619\,89 + 63.411\,032}{2}$$
$$\doteq 127.515\,461$$
$$\doteq 127.5$$

The period can be calculated from successive maximum.

$$\text{Period} \doteq 0.009\,152\,56 - 0.005\,177\,53$$
$$\doteq 0.003\,975\,03$$
$$\doteq 0.003\,98$$

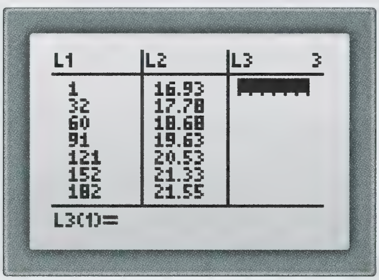
You can also determine the period by calculating $2\pi \div 1581.645\,811$. The resulting period is 0.003 972 561 5.

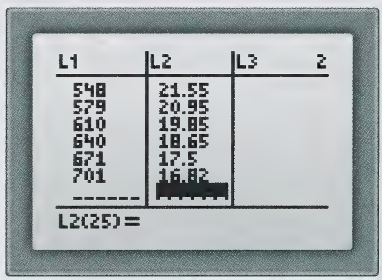
$$\frac{2\pi}{b} \doteq \frac{2\pi}{1581.646\,169}$$
$$\doteq 0.003\,972\,560\,6$$

The amplitude matches a; the median matches d; and the period matches $\frac{2\pi}{b}$.

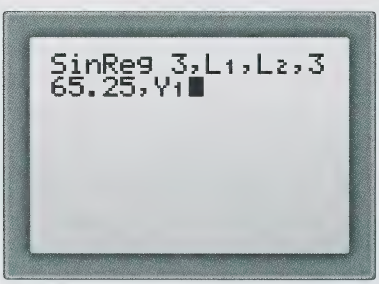
3. Textbook exercise 1 of "Exercises: Checking Your Skills," p. 229

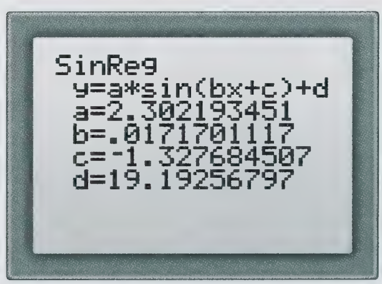
1. a. Enter the day number into list L1 and the time of sunset into list L2.



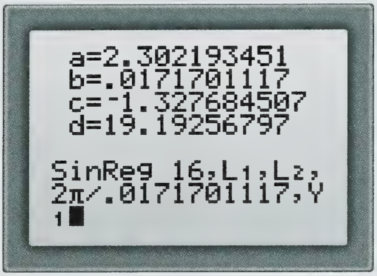


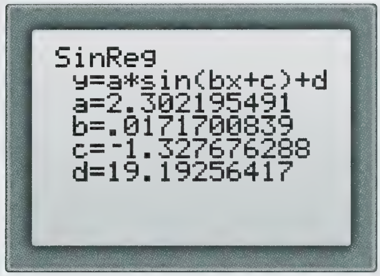
Determine the regression equation.





Obtain a better-fit equation.



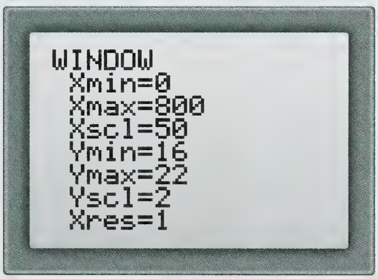


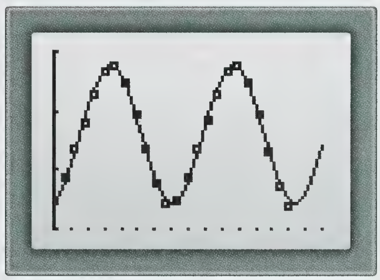
Because the values are only slightly different from the initial regression, there is no need to find any further better-fit regression equations.

Therefore, the regression equation is $y \doteq 2.302 \sin\left(0.0172x - 1.328\right) + 19.192$.

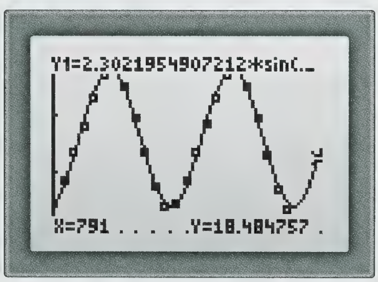
b. Method 1: Using the Graph

Graph the data and the regression equation. Be sure to set the window settings to include March 1, 2001, which is day $701 + 31 + 31 + 28 = 791$.





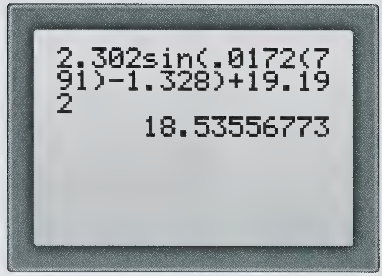
Use the Value feature from the CALCULATE menu to predict the time of sunset for March 1, 2001.



The time of sunset for March 1, 2001, will be about 18.48 h, or 6:29 P.M.

Method 2: Using the Regression Equation

Substitute 791 for x into the regression equation.



$$
\begin{aligned}
y &\doteq 2.302 \sin\left(0.0172x - 1.328\right) + 19.192 \\
&\doteq 2.302 \sin\left[0.0172(791) - 1.328\right] + 19.192 \\
&\doteq 18.535\,567\,73 \\
&\doteq 18.54
\end{aligned}
$$

The time of sunset for March 1, 2001, will be about 18.54 h, or 6:32 P.M.

c. The median time of sunset is the average of the maximum and minimum times of sunset. The value of d in the regression equation can be used for this as well. Therefore, the median time of sunset would be about 19.19 h, or about 7:12 P.M.

d. The maximum time of sunset varies from the median time by the amplitude. The amplitude is represented by a in the regression equation (2.302 195 491). Therefore, the maximum time of sunset varies from the median time by about 2.30 h.

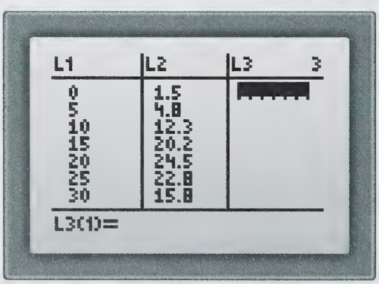
4. Textbook exercises 3 and 4 of "Discussing the Ideas," p. 229

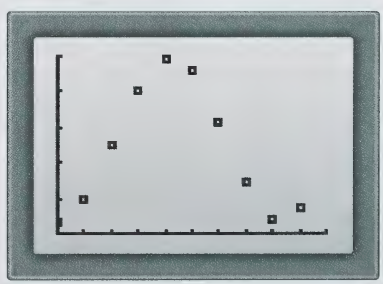
3. The results of SinReg are coefficients of an equation of the form $y = a \sin\left(bx + c\right) + d$. You need to relate these numbers to the problem at hand. Things like period, maximum, and minimum are easily found from the coefficients if you interpret them correctly.

4. The regression equation is a reasonable fit of the data, but far from a perfect fit. This means the predictions made with the regression equation will not be perfect, but they should not be that far from actual values.

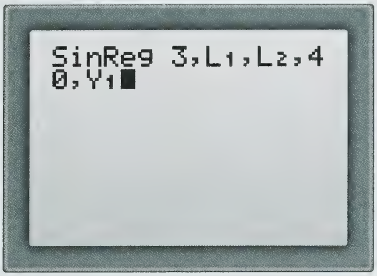
5. Textbook exercises 4 and 5 of "Exercises: Checking Your Skills," p. 231

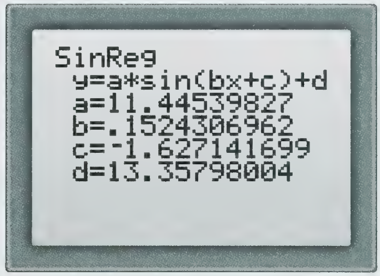
4. a. Enter and graph the data.



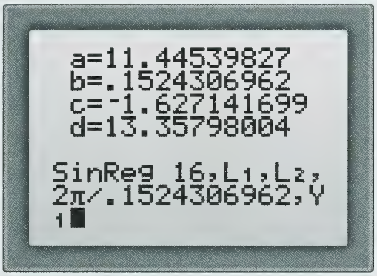


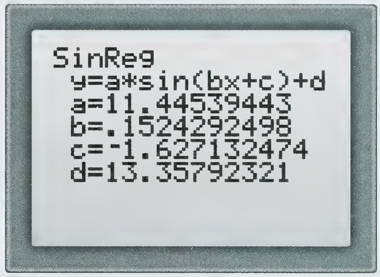
The period appears to be about $40-0=40$ s. Use it to determine the initial regression equation.





Obtain a better-fit regression equation.

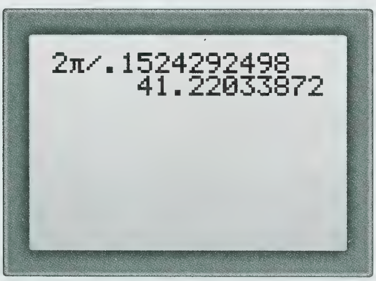




Because the values are only slightly different from the initial regression equation, there is no need to determine any further better-fit equations.

Therefore, the regression equation is $y \doteq 11.445 \sin\left(0.152x-1.627\right)+13.358$.

b.



This method is used to determine the period because you don't have two distinct maximums or minimums.

The period is about 41.22 s.

c. Maximum height = Median height + Amplitude

$$\begin{aligned} &= d + a \\ &\doteq 13.3579 + 11.4454 \\ &\doteq 24.8037 \\ &\doteq 24.80 \end{aligned}$$

The maximum height above the ground is about 24.80 m.

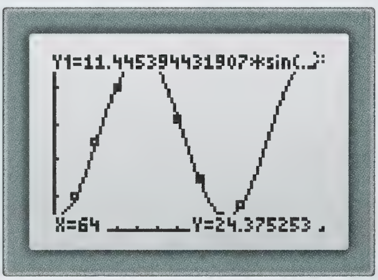
d. The diameter is twice the amplitude.

$$\begin{aligned} \text{Diameter} &\doteq 2 \times 11.4454 \\ &\doteq 22.8908 \\ &\doteq 22.89 \end{aligned}$$

The diameter of the Ferris wheel is about 22.89 m.

e. **Method 1: Using the Graph**

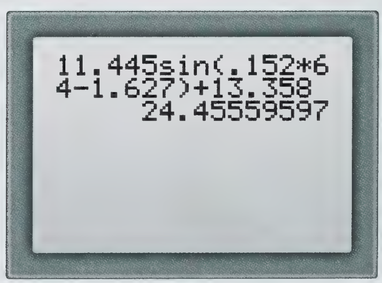
Extend the window settings to include $x = 64$. Then use the Value feature from the CALCULATE menu to determine the height.



The seat will be about 24.38 m high.

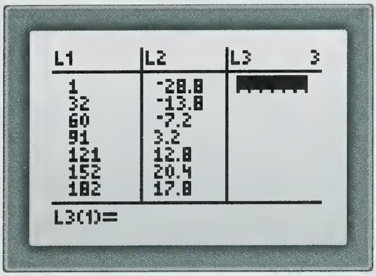
Method 2: Using the Regression Equation

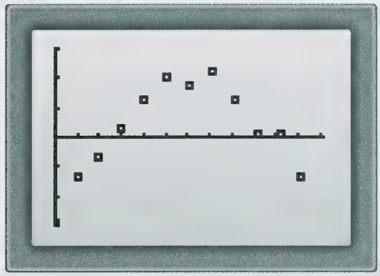
$$\begin{aligned} y &\doteq 11.445 \sin(0.152x - 1.627) + 13.358 \\ &\doteq 11.445 \sin[0.152(64) - 1.627] + 13.358 \\ &\doteq 24.455\,595\,97 \\ &\doteq 24.46 \end{aligned}$$



The seat will be about 24.46 m high.

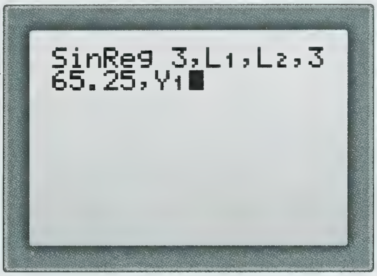
5. **a.**

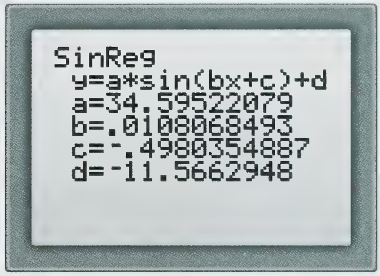




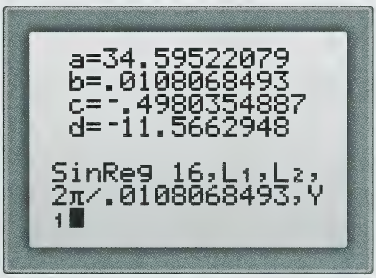
This is similar to a complete cycle of a sinusoidal wave; thus, the period is about 1 year.

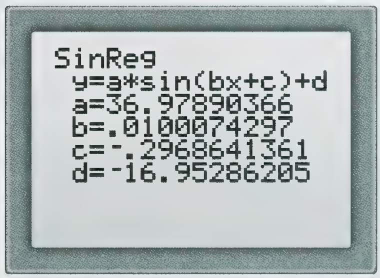
b.



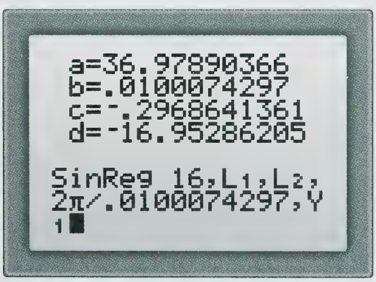


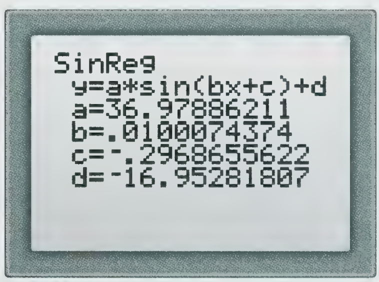
Obtain a better-fit equation.





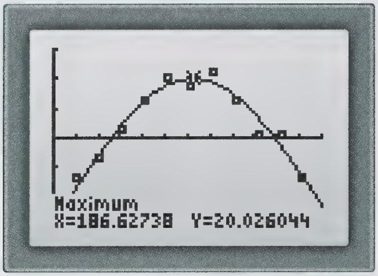
Because the values are significantly different from the initial regression equation, there is a need to determine a better-fit equation.





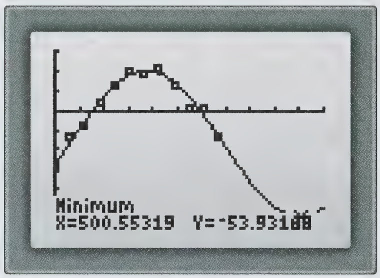
Because the values are only slightly different from the previous regression equation, there is no need to determine any further better-fit equations.

Therefore, the regression equation is $y \doteq 36.979 \sin(0.0100x - 0.297) - 16.953$.

c. 

The highest maximum temperature is about 20.0°C. This temperature occurs on about the 187th day of the year, or July 5, 1999.

Extend the window settings to determine the minimum.



According to the regression, the minimum temperature is about $-53.9°$. This temperature is predicted to occur on about the 501st day, or May 15, 2000. **Note:** Discrepancies between these answers and the answers in the textbook are due to the refinement of the regression equation.

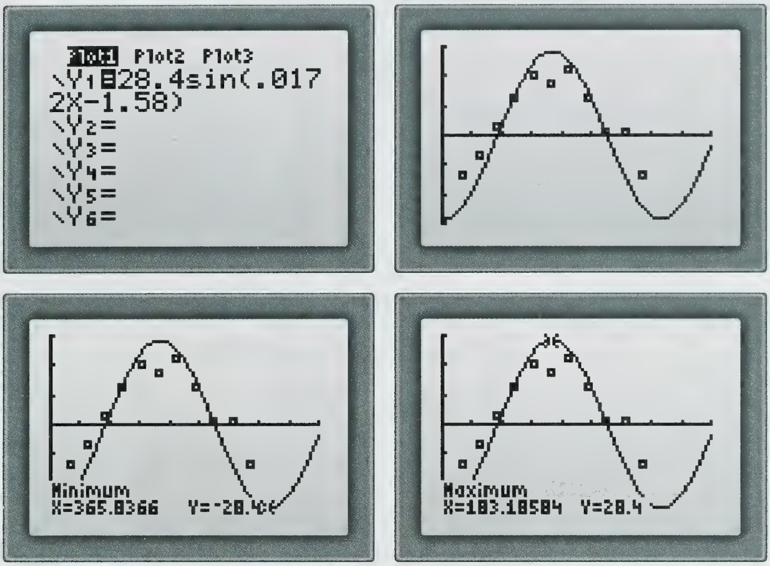
d. The median yearly maximum temperature is about $-17.0°\text{C}$ (the value of *d* in the equation).

e. The maximums vary from the median maximum by about 36.98°C (the value of *a* in the equation).

f. While the regression equation is the best fit to the data that the TI-83 could find, it is a poor fit to reality.

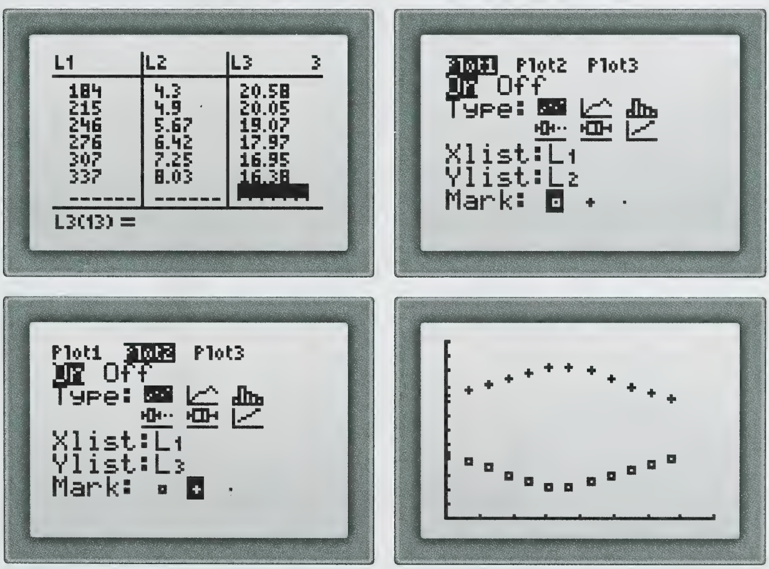
When using technology to assist with solutions to questions, there is always the danger that you will let common sense be overruled by the belief that the machine (in this case the calculator) is always correct. You should view the results of regressions with some skepticism unless they can be verified in some other way. The answers for 5.c., 5.d., and 5.e. are considerably different than reality.

The following shows another sinusoidal equation and graph that fits reality better, but it has poorer least-mean-squares results.



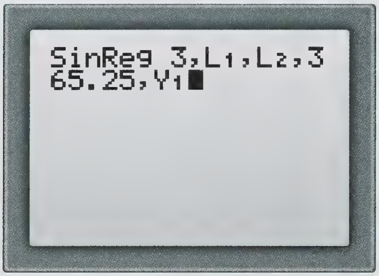
6. Textbook exercise 6 of Exercises: Extending Your Thinking," p. 232

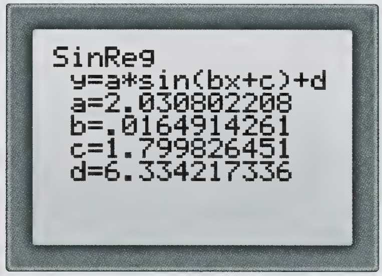
6. a. Place the day number in list L1, the sunrise time in list L2, and the sunset time in list L3. The scatterplots should look like the following.

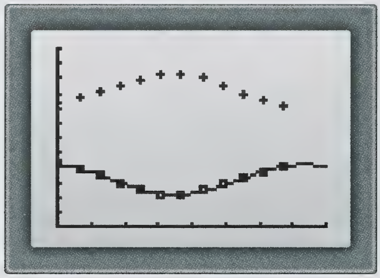


Xmin = 0
Xmax = 400
Xscl = 50
Ymin = 20
Ymax = 24
Yscl = 2
X res = 1

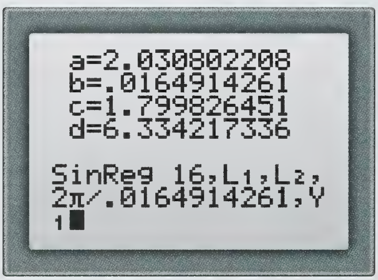
i. The regression for sunrise times can be calculated as follows.

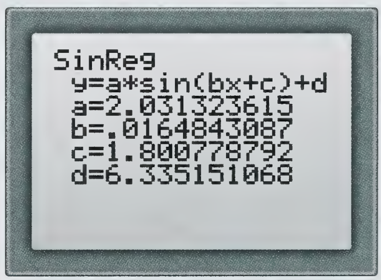


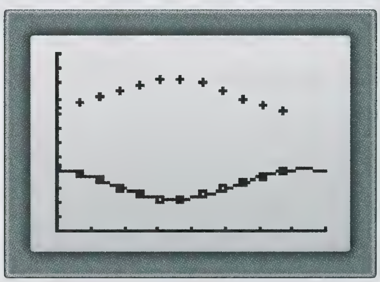




Obtain a better-fit regression equation.



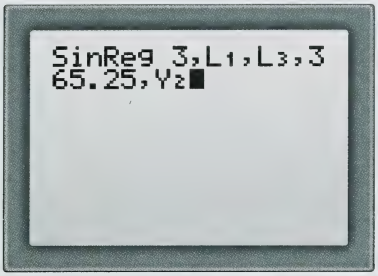


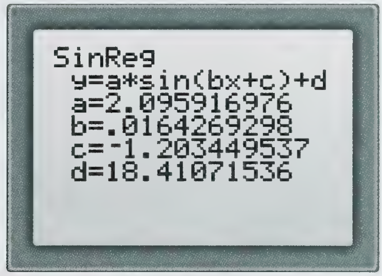


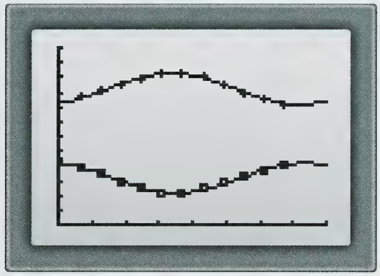
There is no need to determine any further better-fit equations.

Therefore, the regression equation is $y \doteq 2.031 \sin\left(0.016x + 1.801\right) + 6.335$.

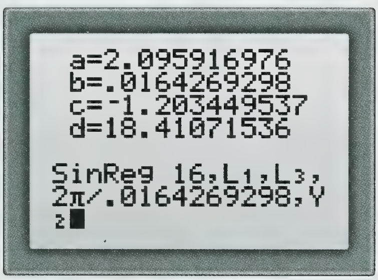
ii. The regression for sunset times can be calculated as follows.

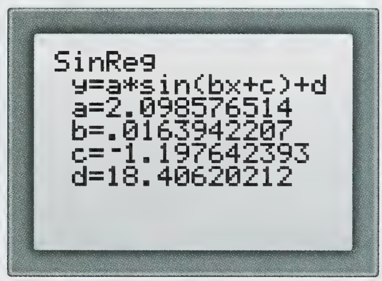


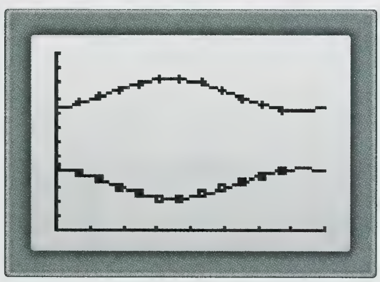




Obtain a better-fit regression equation.







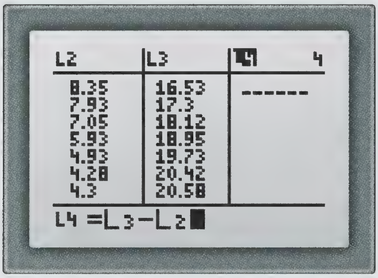
There is no need to determine any further better-fit equations.

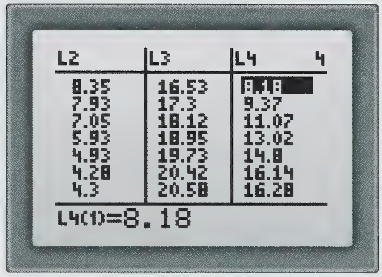
Therefore, the regression equation is $y \doteq 2.099 \sin\left(0.016\,x - 1.198\right) + 18.406$.

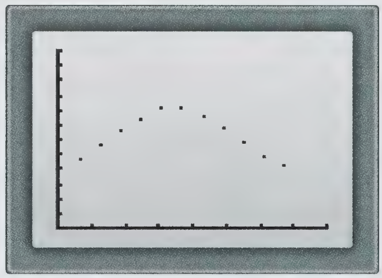
iii. The regression for number of hours of sunlight can be determined by subtracting list L2 from list L3. Place the difference in list L4.

Place the cursor over "L4" and press

2nd [L3] − 2nd [L2] ENTER

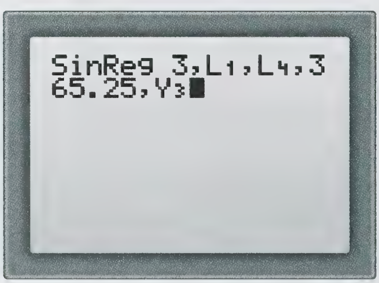


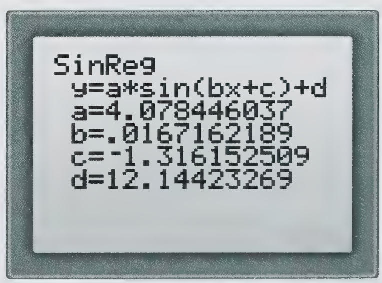


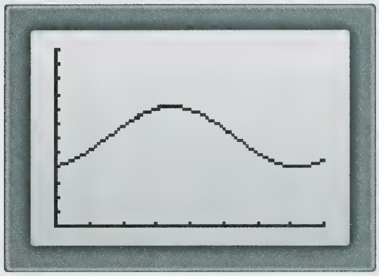


Note: Turn off Plots 1 and 2 as well as the graphs of Y_1 and Y_2 so only Plot 3 will be displayed. To turn off the graphs of Y_1 and Y_2, place the cursor over the equal sign and press ENTER. This will remove the black square over the equal sign.

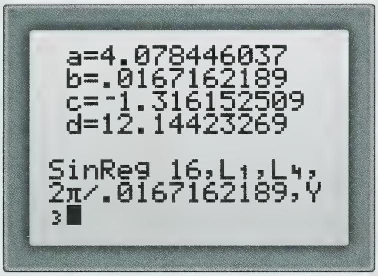
Determine the initial regression equation.

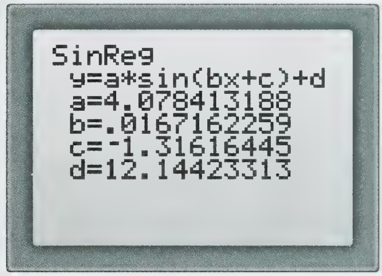


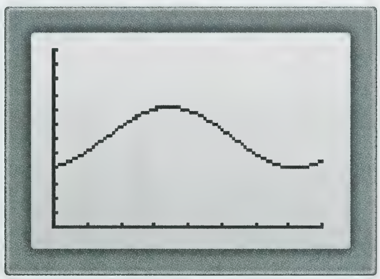




Obtain a better-fit equation.







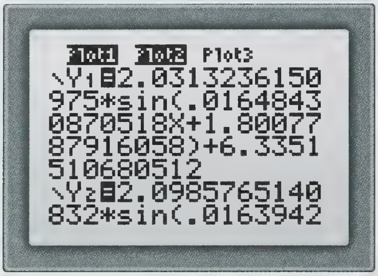
There is no need to determine any further better-fit equations.

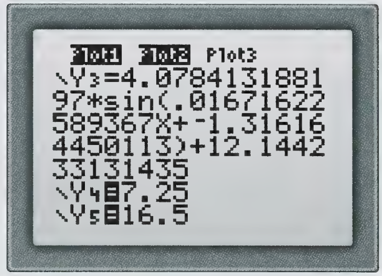
Therefore, the regression equation is $y \doteq 4.078 \sin\left(0.017x - 1.316\right) + 12.144$.

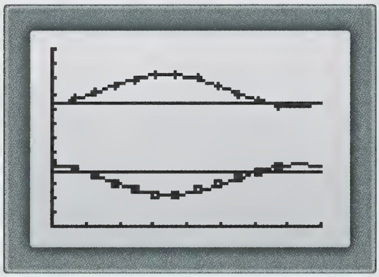
b. Answers may vary. A sample answer is given.

The assumptions will include the belief that the regression equations give actual results, that the schools use the same time (such as not changing to daylight savings time at a critical point relative to the information used in the question), and that students don't go to school every day.

Start by graphing the regression equations for sunrise and sunset along with the lines showing 7:15 A.M. $\left(y = 7.25\right)$ and 4:30 P.M. $\left(y = 16.50\right)$.

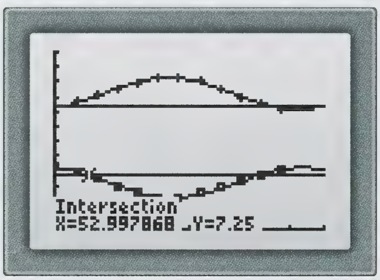


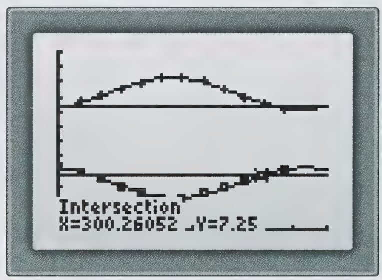


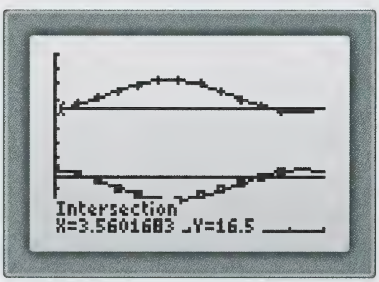


Notice that Plot 3 and Y_3 have been deselected so they do not appear in the graph.

Use the Intersect feature from the CALCULATE menu to find where these lines cross the regression equations.







Students will be starting their day in the dark from the start of the year to day 53 (February 22). They will then start the day in the light until day 300 (October 26).

Students will end their day in the dark from the start of the year to day 4 (January 4). They will then end the day in the light until day 334 (November 29).

The period when the students are in the dark at the end of the day is included in the period when they are in the dark at the start of the day. There will be 53 days at the start of the year and 66 days (2000 is a leap year) at the end of the year when there is the possiblility of school bus rides being in the dark. Since students don't generally go to school on Saturdays or Sundays these 119 days should be reduced by about 33 days for weekends. The remaining 86 days will have to be reduced for other non-school days, like holidays and spring break. In most school divisions these account for another 10 days, leaving 76 days when students spend part of their time riding the school buses in the dark.

7. Project Book exercises, p. 82

- Temperatures will vary considerably as exposure to high heat or cold when the tide is out will greatly affect the temperature. When the tide is in, the water will have a moderating effect on temperature conditions for the marine life. Amount of light will be affected in areas where the water becomes relatively deep with the incoming tide. Moisture conditions can be greatly affected for marine life that exists on the surface when the tide is out.

- Animals that live on the shore include mussels, seaweed, sea anenome, starfish, crabs, oysters, crayfish, clams, flatworms, sea urchins, sponges, and barnacles to name a few.

- Animals may adapt to the changing environment by burrowing in the sand when the tide is out, sticking to rocks, having camouflage, or hiding when the tide is out. Plants may develop tough leaves to resist the abrasive action of the tides.

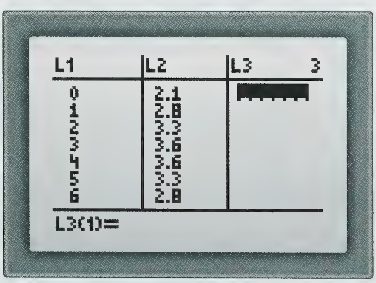
- Some animals have air bladders so they stay close to the surface and to adequate sunlight when the tide is in. Some animals stick to the rocks to avoid being swept out by the waves. Some plants develop holdfasts to anchor themselves to the rocks.

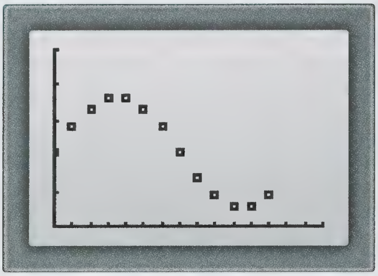
- Sedentary animals that are out of the water only a few hours might adapt by burrowing in the sand or hiding under some object.

- Types of pollution that might be seen on the ocean's shoreline are oil slicks, floating garbage, warm water from an industrial site, and chemical pollutants to name a few.

8. Project Book exercises 1, 2, and 3 of "Getting Started," p. 83

1.



2. 

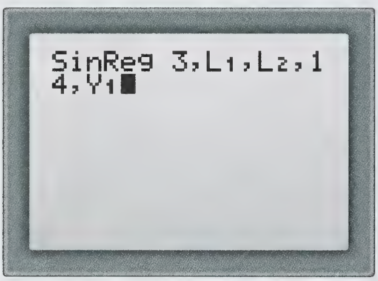
High tide appears to occur between 3:00 and 4:00.

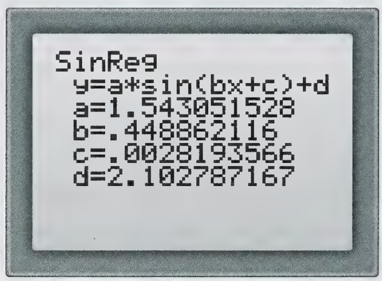
$$\text{Amplitude} = \frac{\text{Maximum value} - \text{Minimum value}}{2}$$
$$= \frac{3.6 - 0.6}{2}$$
$$= 1.5 \text{ feet}$$

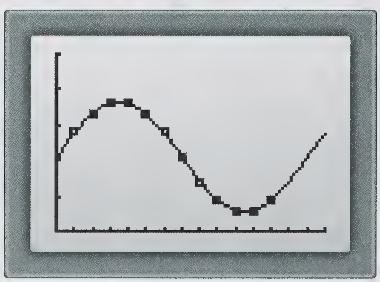
The amplitude is the amount the tide goes above and below the median tide level.

3. The period of the graph appears to be about 14 h.

Obtain the initial regression equation.

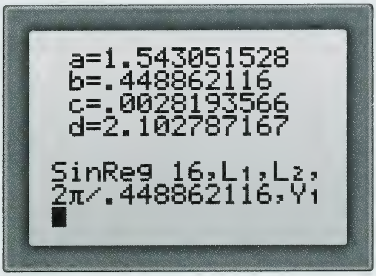


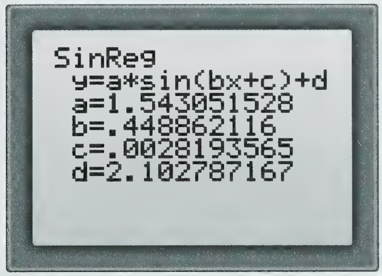


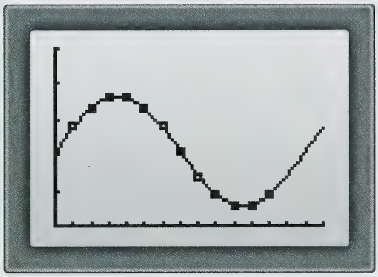


Make sure the window settings are set so at least one complete cycle is visible.

Obtain a better-fit regression equation.

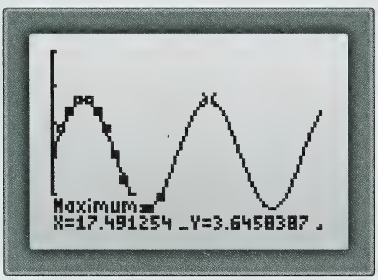


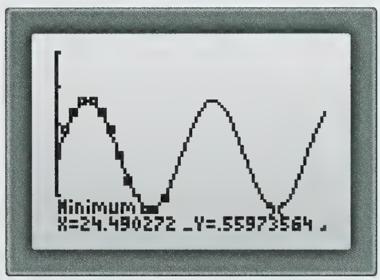




There is no need to determine any further better-fit equations.

Extend the window settings so the graph shows two complete cycles. Use the Maximum and Minimum features from the CALCULATE menu to determine the first high tide and the first low tide.





The next high tide will occur at about 17.49 h, or 5:29 P.M.; the next low tide will occur at about 24.49 h, or about 12:29 A.M.

9. An oceanographer collects physical information and data about the ocean or areas near the ocean and develops a mathematical model of the tides. A marine biologist collects information and data about animals and plants that live in the ocean or on the shore and uses a mathematical model of the tides in the analysis of the information about the animals and plants.

10. Textbook exercise "Communicating the Ideas," p. 232

Using a curve of best fit allows you to make predictions about events where you don't have data. If the best-fit curve describes the events realistically, the predictions will be quite close to reality. The down side is that if the curve doesn't describe the events realistically, the predictions could be wildly incorrect.

Activity 4: The Characteristics of $y = a\sin(bx + c) + d$

1. a. When the start point (orange ✚) is dragged upward, the value of D in the equation increases and the entire graph moves upward on the coordinate plane. At point $(0, 1)$ the value of d is 1.

   When the start point is dragged downward, the value of D in the equation decreases and the entire graph moves downward on the coordinate plane. At point $(0, -1)$ the value of d is -1.

   When the start point is returned to the origin, the value of D returns to 0 and the graph is centered vertically at the origin.

   b. When the start point is dragged toward the left, the value of C increases and the entire graph moves toward the left. At the point $(-\pi, 0)$, the value of c is about 3.14.

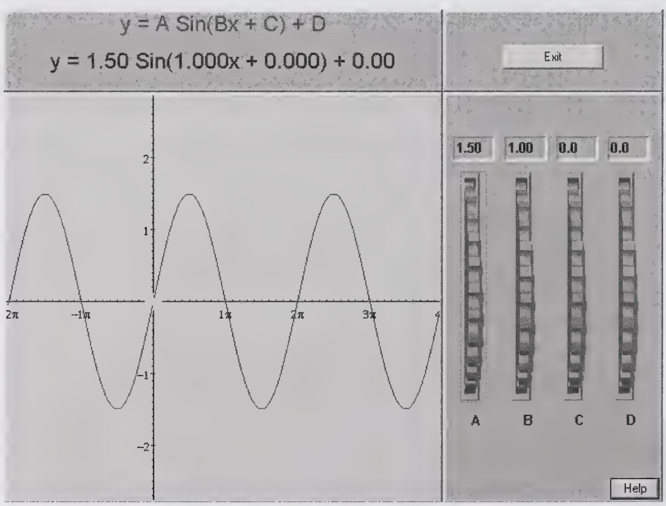
   When the start point is dragged toward the right, the value of C decreases and the entire graph moves toward the right. At the point $(\pi, 0)$, the value of C is about -3.14.

   When the start point is returned to the origin, the value of C returns to 0.

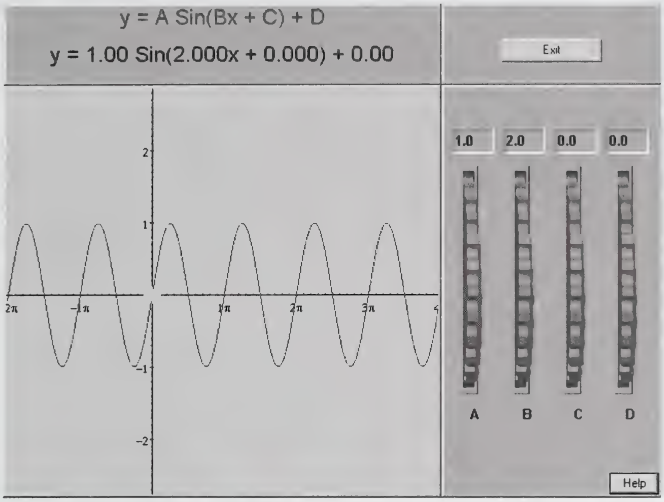
   c. When a point is dragged upward, the graph grows vertically and the value of A increases. When a point is dragged downward, the graph shinks vertically and the value of A decreases.

   d. When a point is dragged to the left, the period of the graph decreases and the value of B increases. When a point is dragged to the right, the period of the graph increases and the value of B decreases.

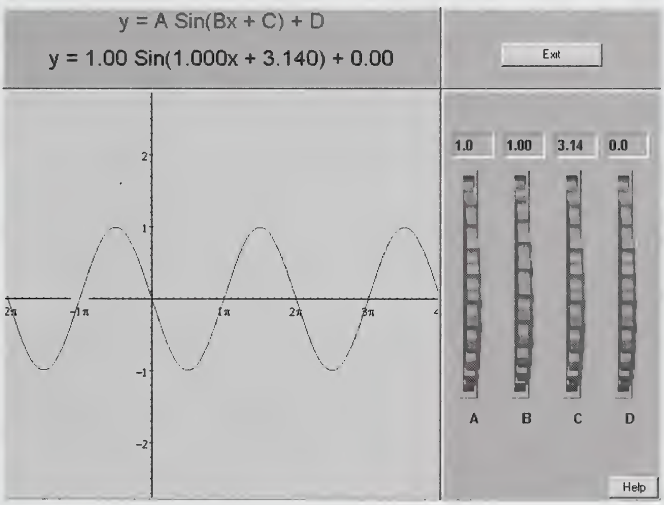
2. a. When the value of A changes, the amplitude changes. If the value of A increases, the amplitude increases (expands vertically), and vice versa. For example, when the value of A changes from 1 to 1.5, the amplitude changes from 1 to 1.5.



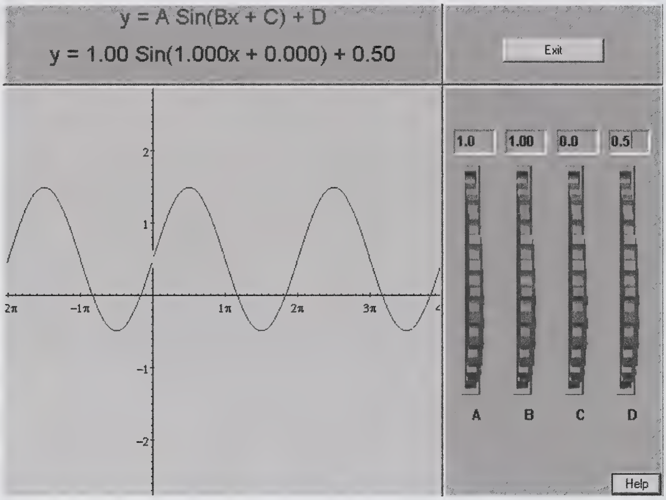
b. When the value of B changes, the period changes. If the value of B increases, the period decreases, and vice versa. For example, when the value of B changes from 1 to 2, the period changes from 2π to π.



c. When the value of C changes, the graph either moves to the left or to the right. If $C > 0$, the graph shifts to the left the corresponding amount. If $C < 0$, the graph shifts to the right the corresponding amount. For example, when the value of C changes from 0.0 to 3.14, the graph shifts to the left 3.14 radians.



d. When the value of D changes, the graph either moves up or down. If $D > 0$, the graph moves up the corresponding amount. If $D < 0$, the graph moves down the corresponding amount. For example, when the value of D changes from 0.0 to 0.5, the graph moves up 0.5 units.

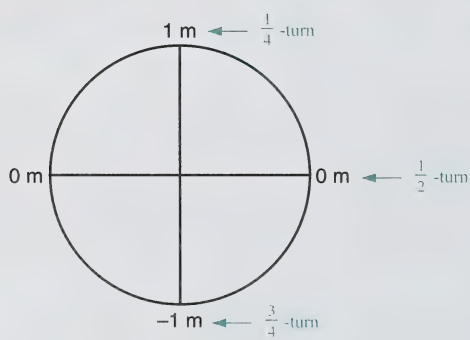


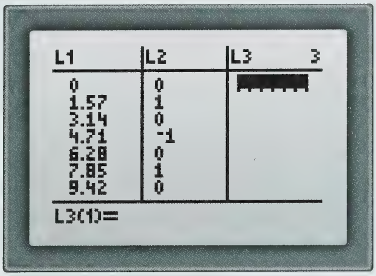
e. **i.** The coefficient A affects the amplitude of the graph.
ii. The coefficient B affects the period of the graph.
iii. The coefficients C and D affect the starting point of the graph.
iv. The coefficient D affects the vertical displacement of the graph.

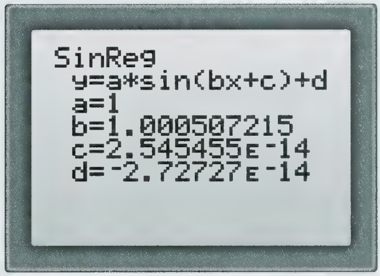
3. Textbook exercises 1 to 15 of "Investigation: Exploring the Effect of the Parameters *a*, *b*, *c*, and *d*," pp. 233 to 236

1.

Revolution	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Time (s)	0	1.57	3.14	4.71	6.28	7.85	9.42	10.99	12.56
Height	0	1	0	−1	0	1	0	−1	0

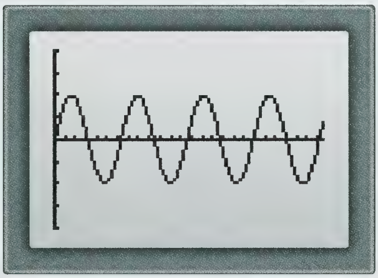


2. 



The regression equation, rounded to 2 decimal places in each coefficient, is $y \doteq 1.00 \sin(1.00x + 0.00) + 0.00$, or more simply as $y = \sin x$.

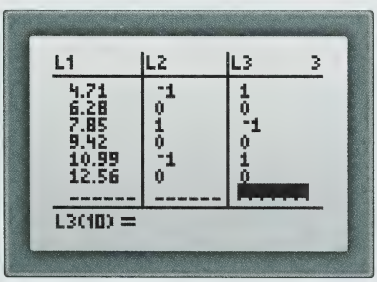
3. The graph of the regression equation should look like the following.

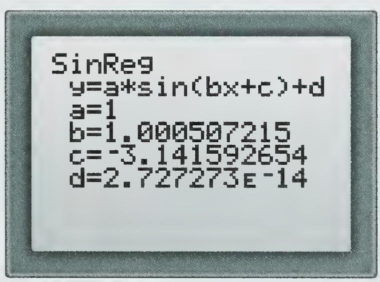


4.

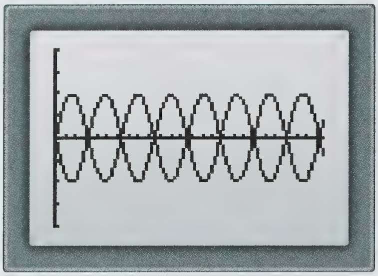
Revolution	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Time (s)	0	1.57	3.14	4.71	6.28	7.85	9.42	10.99	12.56
Height	0	−1	0	1	0	−1	0	1	0

5. Enter the height in list L3.





The regression equation, with coefficients rounded to 2 decimal places, is $y \doteq 1.00 \sin(1.00x - 3.14) + 0.00$, or more simply as $y = \sin(x - 3.14)$.

6. 

7. a. $$\frac{\text{Maximum value} - \text{Minimum value}}{2} = \frac{1-(-1)}{2}$$
$$= \frac{2}{2}$$
$$= 1$$

The amplitude is 1.

b. The period is 6.28.

c. $$\frac{\text{Maximum value} + \text{Minimum value}}{2} = \frac{1+(-1)}{2}$$
$$= \frac{0}{2}$$
$$= 0$$

The median value is 0.

d. The maximum value is 1, and the minimum value is -1.

e. The x-coordinate of the point on the increasing part of the curve closest to the y-axis whose coordinate is the median value is 3.14 .

8. a. The parameter a is 1 and matches the amplitude.

b. $$\frac{2\pi}{b} \doteq \frac{6.28}{1}$$
$$\doteq 6.28$$

This value is the period.

c. The parameter d is 0 and matches the median value.

d. $d+a=1$ and $d-a=-1$

These values match the maximum and minimum values respectively.

e. $-\frac{c}{b} \doteq -\frac{-3.14}{1}$

$\doteq 3.14$

This value matches the graph's horizontal shift from 0.

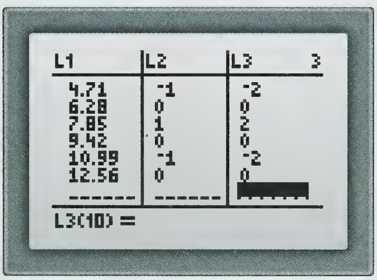
9. The equations differ only in the value of c. The graphs from exercises 2 and 5 have the same period and general shape, but the second graph is shifted to the right 3.14 units. This matches the change in the value of c.

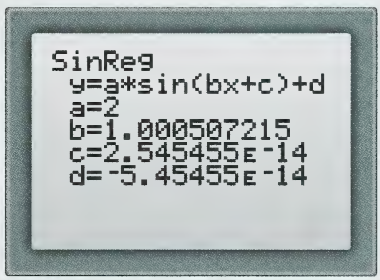
10.

Revolution	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Time (s)	0	1.57	3.14	4.71	6.28	7.85	9.42	10.99	12.56
Height	0	2	0	−2	0	2	0	−2	0

11. Repeat exercises 5 to 9.

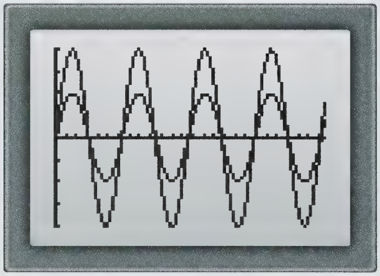
5.





The regression equation, with coefficients rounded to 2 decimal places, is $y \doteq 2.00 \sin\left(1.00x+0.00\right)+0.00$, or more simply as $y=2\sin x$.

6. The graphs of both equations look like the following.



7. a. $$\frac{\text{Maximum value} - \text{Minimum value}}{2} = \frac{2-(-2)}{2} = \frac{4}{2} = 2$$

The amplitude is 2.

b. The period is 6.28.

c. $$\frac{\text{Maximum value} + \text{Minimum value}}{2} = \frac{2+(-2)}{2} = \frac{0}{2} = 0$$

The median value is 0.

d. The maximum value is 2, and the minimum value is -2.

e. The x-coordinate of the point on the increasing part of the curve closest to the y-axis whose y-coordinate is a median value is 0.

8. a. The parameter a is 2 and matches the amplitude.

b. $$\frac{2\pi}{b} \doteq \frac{6.28}{1} \doteq 6.28$$

This value is the period.

c. The parameter d is 0 and matches the median value.

d. $d+a=2$ and $d-a=-2$

These values match the maximum and minimum values respectively.

e. $-\frac{c}{b} \doteq -\frac{0}{1}$

$\doteq 0$

This value matches the graph's horizontal shift from 0.

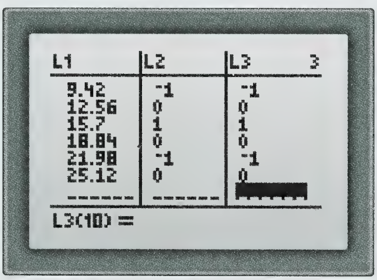
9. The equations differ only in the value of a. The graphs from exercises 2 and 11 have the same period, shape, and zero points. The second graph is enlarged vertically by a factor of 2. This matches the change in the value of a.

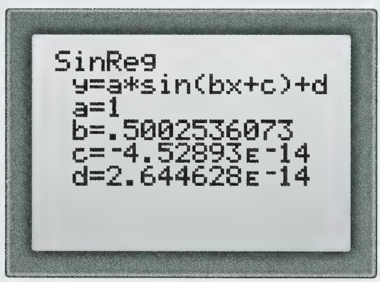
12.

Revolution	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Time (s)	0	3.14	6.28	9.42	12.56	15.70	18.84	21.98	25.12
Height	0	1	0	−1	0	1	0	−1	0

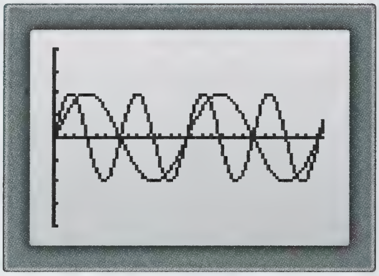
13. Repeat exercises 5 to 9.

5.





The regression equation, with coefficients rounded to 2 decimal places, is $y \doteq 1.00 \sin\left(0.50x-0.00\right)+0.00$, or more simply as $y \doteq \sin\left(0.5x\right)$.

6. 

7. a. $$\frac{\text{Maximum value} - \text{Minimum value}}{2} = \frac{1-(-1)}{2}$$
$$= \frac{2}{2}$$
$$= 1$$

The amplitude is 1.

b. The period is 12.56.

c. $$\frac{\text{Maximum value} + \text{Minimum value}}{2} = \frac{1+(-1)}{2}$$
$$= \frac{0}{2}$$
$$= 0$$

The median value is 0.

d. The maximum value is 1, and the minimum value is -1.

e. The x-coordinate of the point on the increasing part of the curve closest to the y-axis whose y-coordinate is a median value is 0.

8. a. The parameter a is 1 and matches the amplitude.

b. $$\frac{2\pi}{b} \doteq \frac{6.28}{0.5}$$
$$\doteq 12.56$$

This value is the period.

c. The parameter d is 0 and matches the median value.

d. $d+a=1$ and $d-a=-1$

These values match the maximum and minimum values respectively.

e. $$-\frac{c}{b} \doteq -\frac{0}{0.5}$$
$$\doteq 0$$

This value matches the graph's horizontal shift from 0.

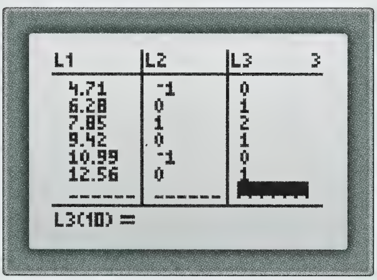
9. The equations differ only in the value of b. The graphs from exercises 2 and 13.5 have different periods, but the same shape and zero points. The second one is enlarged horizontally by a factor of 2. This matches the change in the value of b.

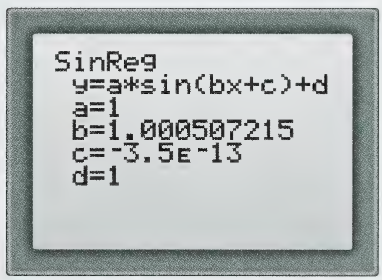
14. The table is completed as follow.

Revolution	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Time (s)	0	1.57	3.14	4.71	6.28	7.85	9.42	10.99	12.56
Height	1	2	1	0	1	2	1	0	1

15. Repeat exercises 5 to 9.

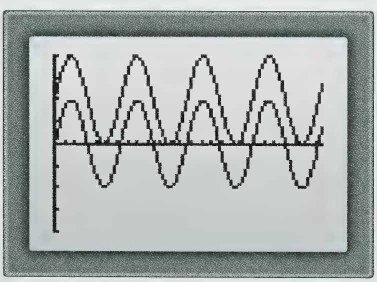
5.





The regression equation, with coefficients rounded to 2 decimal places, is $y \doteq 1.00 \sin\left(1.00x+0.00\right)+1.00$, or more simply as $y \doteq \sin x + 1$.

6.



7. a. $$\frac{\text{Maximum value} - \text{Minimum value}}{2} = \frac{2-0}{2}$$
$$= \frac{2}{2}$$
$$= 1$$

The amplitude is 1.

b. The period is 6.28.

c. $$\frac{\text{Maximum value} + \text{Minimum value}}{2} = \frac{2+0}{2}$$
$$= \frac{2}{2}$$
$$= 1$$

The median value is 1.

d. The maximum value is 2, and the minimum value is 0.

e. The x-coordinate of the point on the increasing part of the curve closest to the y-axis whose y-coordinate is a median value is 0.

8. a. The parameter a is 1 and matches the amplitude.

b. $$\frac{2\pi}{b} \doteq \frac{6.28}{1}$$
$$\doteq 6.28$$

This value is the period.

c. The parameter d is 1 and matches the median value.

d. $d + a = 2$ and $d - a = 0$

These values match the maximum and minimum values respectively.

e. $$-\frac{c}{b} \doteq -\frac{0}{1}$$
$$\doteq 0$$

This value matches the graph's horizontal shift from 0.

9. The equations differ only in the value of d. The graphs from exercises 2 and 15.5 have the same periods, shape, and zero points. The second one is raised vertically by a factor of 1. This matches the change in the value of d.

4. a. **Textbook exercises 1 and 2 of "Discussing the Ideas," p. 238**

1. The parameter a can be estimated by half the difference between the largest value on the graph and the smallest value on the graph.

   The parameter b can be estimated from the period by using the equation $b=\frac{2\pi}{\text{Period}}$.

   The parameter c can be estimated by how far from the origin the sine graph starts and then using the equation $c=-b\times\text{offset}$.

   The parameter d can be estimated by the distance from the x-axis to the median of the graph.

2. The amplitude and median are both related to the maximum and minimum values. The amplitude is half the difference of these values, and the median is half the sum of these values. The amplitude shows how far from the median value the sinusoidal function moves.

b. **Textbook exercises 1.a., 1.b., 1.d., 2.a., 2.d., 3, 5, and 6 of Exercises: Checking Your Skills," pp. 238 to 240**

1. a. i. Maximum value $= d+a$

$$\begin{aligned} &= -1+2 \\ &= 1 \end{aligned}$$

ii. Minimum value $= d-a$

$$\begin{aligned} &= -1-2 \\ &= -3 \end{aligned}$$

iii. The amplitude, a, is 2.

iv. Period $=\frac{2\pi}{b}$

$$\begin{aligned} &= \frac{2\pi}{1} \\ &= 2\pi \end{aligned}$$

v. The median value, d, is -1.

vi. Start point $=\left(-\frac{c}{b}, d\right)$

$$\begin{aligned} &= \left(-\frac{0}{1}, -1\right) \\ &= (0, -1) \end{aligned}$$

Activity 4 (continued)

b. **i.** Maximum value $= d + a$
$= 0 + 1$
$= 1$

ii. Minimum value $= d - a$
$= 0 - 1$
$= -1$

iii. The amplitude, a, is 1.

iv. Period $= \dfrac{2\pi}{b}$
$= \dfrac{2\pi}{2}$
$= \pi$

v. The median value, d, is 0.

vi. Start point $= \left(-\dfrac{c}{b}, d\right)$
$= \left(-\dfrac{\frac{\pi}{3}}{2}, 0\right)$
$= \left(\dfrac{\pi}{6}, 0\right)$

d. **i.** Maximum value $= d + a$
$= -0.5 + 0.66$
$= 0.16$

ii. Minimum value $= d - a$
$= -0.5 - 0.66$
$= -1.16$

iii. The amplitude, a, is 0.66.

iv. Period $= \dfrac{2\pi}{b}$
$= \dfrac{2\pi}{2}$
$= \pi$

v. The median value, d, is -0.5.

vi. Start point $= \left(-\dfrac{c}{b}, d\right)$
$= \left(-\dfrac{0}{2}, -0.5\right)$
$= (0, -0.5)$

2. **a.** The maximum value is 3.

The minimum value is -3.

$$\begin{aligned}\text{Amplitude} &= \frac{\text{Maximum} - \text{Minimum}}{2} \\ &= \frac{3-(-3)}{2} \\ &= \frac{6}{2} \\ &= 3\end{aligned}$$

The period is about 6.2 or 2π.

$$\begin{aligned}\text{Median value} &= \frac{\text{Maximum} + \text{Minimum}}{2} \\ &= \frac{3+(-3)}{2} \\ &= \frac{0}{2} \\ &= 0\end{aligned}$$

The start point is $(0, 0)$.

d. The maximum value is 1.

The minimum value is -1.

$$\begin{aligned}\text{Amplitude} &= \frac{\text{Maximum} - \text{Minimum}}{2} \\ &= \frac{1-(-1)}{2} \\ &= \frac{2}{2} \\ &= 1\end{aligned}$$

The period is about 6.2 or 2π.

$$\begin{aligned}\text{Median value} &= \frac{\text{Maximum} + \text{Minimum}}{2} \\ &= \frac{1+(-1)}{2} \\ &= \frac{0}{2} \\ &= 0\end{aligned}$$

The start point is about $(-0.8, 0)$.

3. Answers may vary. A sample answer is given.

Start by stretching the graph of $y = \sin x$ vertically by a factor of 3 to obtain the graph of $y = 3 \sin x$. Next, compress the period of $y = 3 \sin x$ by a factor of 2 to obtain the graph of $y = 3 \sin(2x)$. (The period is now π.) Finally, move the graph of $y = 3 \sin(2x)$ down 1 unit to obtain the graph of $y = 3 \sin(2x) - 1$.

5. Answers may vary. A sample answer is given.

The graph is decreasing in height over time, and the length of each cycle is shrinking. By just listening to the guitar, you would hear the sound fading away and maybe notice an increase in pitch.

6. Answers may vary. Sample answers are given.

   a. $$\text{Amplitude} = \frac{\text{Maximum} - \text{Minimum}}{2}$$
   $$= \frac{18-(-25)}{2}$$
   $$= \frac{43}{2}$$
   $$= 21.5$$

   b. The period is 1 year, or 365.25 days.

   c. The mean maximum temperature appears to be 18°C, and the mean minimum temperature appears to be $-25°\text{C}$.

   d. $$\text{Median temperature} = \frac{\text{Maximum} + \text{Minumum}}{2}$$
   $$\doteq \frac{18+(-25)}{2}$$
   $$\doteq \frac{-7}{2}$$
   $$\doteq -3.5°\text{C}$$

   e. Each mark on the *x*-axis is about 31 days. The maximum temperature occurs at about the 190th day of the year, or approximately July 9.

   f. The minimum value seems to be just after the start of the year, approximately January 4.

5. Textbook exercise "Communicating the Ideas," p. 240

Answers may vary. A sample answer is given.

The parameter a is the amplitude of the graph. It shows how far the values vary from the median value, d.

The parameter b is used to determine the period of the graph using the formula:

$$\text{Period} = \frac{2\pi}{b}$$

The parameters b, c, and d are used to determine how far the graph's starting point has shifted from the origin.

$$\left(-\frac{c}{b}, d\right)$$

Activity 5: Applications Involving Sinusoidal Data

1. a. **Textbook exercise 1 of "Discussing the Ideas," p. 244**

 1. Answers may vary. A sample answer is given.

 In North America, the electrical utility companies supply electricity to homes and businesses that have a voltage variation with a period of $\frac{1}{60}$ s. Other parts of the world, like England, have electrical systems that use a period of $\frac{1}{50}$ s.

 b. **Textbook exercises 2 and 5 of "Exercises: Checking Your Skills," p. 245 and 246**

 2. a. Answers may vary slightly.

 The graph has a maximum value of 4.0 and minimum value of 2.0.

$$\begin{aligned}\therefore \text{ Amplitude} &= \frac{\text{Maximum} - \text{Minimum}}{2} \\ &= \frac{4.0-2.0}{2} \\ &= \frac{2.0}{2} \\ &= 1.0\end{aligned}$$

$$\begin{aligned}\therefore \text{ Median} &= \frac{\text{Maximum} + \text{Minimum}}{2} \\ &= \frac{4.0+2.0}{2} \\ &= \frac{6.0}{2} \\ &= 3.0\end{aligned}$$

 Therefore, the value of a is 1.0 and the value of d is 3.0.

 The period of the graph appears to be about 12 h.

$$\begin{aligned}\therefore b &= \frac{2\pi}{\text{Period}} \\ &= \frac{2\pi}{12} \\ &= \frac{\pi}{6}\end{aligned}$$

 The starting point is a point on the rising part of the graph with a y-value of 3.0. This starting point appears to occur at about $(7, 3.0)$.

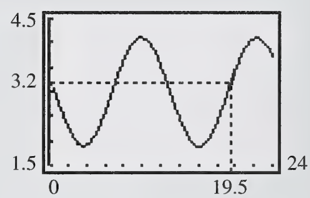
Because $(7, 3.0) = \left(\frac{-c}{b}, d\right)$,

$$\begin{aligned}\frac{-c}{b} &= 7\\ c &= -7b\\ &= -7\left(\frac{\pi}{6}\right)\\ &= -\frac{7\pi}{6}\end{aligned}$$

Therefore, the equation of the graph is

$$\begin{aligned}y &= a \sin(bx + c) + d\\ &\doteq 1.0 \sin\left(\frac{\pi}{6}x - \frac{7\pi}{6}\right) + 3.0\\ &\doteq \sin\left(\frac{\pi}{6}x - \frac{7\pi}{6}\right) + 3.0 \text{ or } \sin(0.52x - 3.67) + 3.0\end{aligned}$$

b. You can estimate the depth from the graph to be about 3.2 m.



The equation found in the answer to exercise 2.a. allows you to estimate the depth of water at 7:30 P.M. by substituting 19.5 h (7:30 P.M.) for x.

$$\begin{aligned}y &\doteq \sin\left(\frac{\pi}{6}x - \frac{7\pi}{6}\right) + 3.0\\ &\doteq \sin\left[\frac{\pi}{6}(19.5) - \frac{7\pi}{6}\right] + 3.0\\ &\doteq 3.25\end{aligned}$$

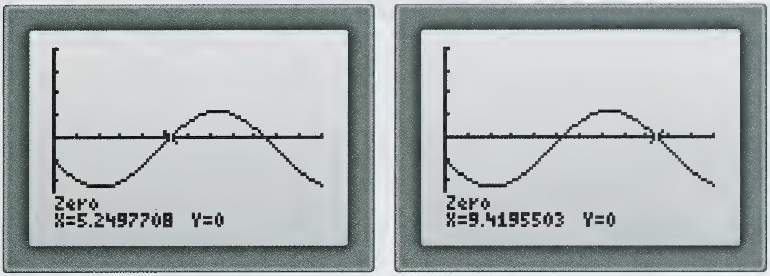
The depth of the water is about 3.25 m at 7:30 P.M.

5. **a.** Since May is the fifth month of the year, the mean daily temperature can be calculated by substituting 5 for x into the equation.

$$\begin{aligned}y &= 17.11 \sin(0.60x - 2.83) - 5.38\\ &= 17.11 \sin[0.60(5) - 2.83] - 5.38\\ &\doteq -2.49\end{aligned}$$

The mean temperature for May is about $-2.49°\text{C}$.

b. Graph the equation and determine when the graph crosses the x-axis. It is between these values that the temperature will be above 0°C. The temperature is above zero for a part of May (month 5); all of June, July, and August; and for a part of September (month 9).



Therefore, the temperatures are higher than 0°C for a period of about 4 months.

2. a. Textbook exercises 2 and 3 of "Discussing the Ideas," p. 244

2. Given the period determined in Example 2, the seasons would indeed shift through the year in an unrealistic way. For example, in 20 years, the temperature cycle would have shifted $20 \times 0.14 = 2.8$ months backward. This would make the warmest month move from July to April and then even earlier in the years to follow.

3. The sun's declination is caused by the tilt of Earth's axis relative to the plane in which it rotates about the sun. Since this tilt is relatively stable, the apparent angle of the sun's rays will vary from perpendicular to the equator by an equal positive and negative amount. This explains the $\pm 23.4°$ variation in the sun's declination.

b. Textbook exercises 1, 3, and 6 of "Exercises: Checking Your Skills," pp. 244 to 247

1. a. The amplitude is the distance the weight is pulled down before being released. This makes the amplitude 0.4 m.

The period is given as 1.2 s.

The median value is the resting position of the weight, which is given as 0.5 m.

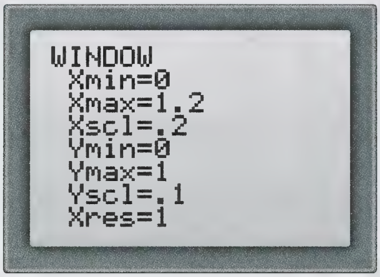
The horizontal translation is one quarter of the period, or 0.3 s.

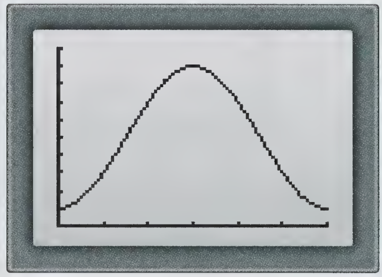
Therefore, the values of the coefficients of the sine equation matching this situation are

$$a = 0.4 \qquad \begin{aligned} b &= \frac{2\pi}{1.2} \\ &\doteq 5.24 \end{aligned} \qquad \begin{aligned} c &= -b \times 0.3 \\ &= -\frac{2\pi}{1.2} \times 0.3 \\ &\doteq -1.57 \end{aligned} \qquad d = 0.5$$

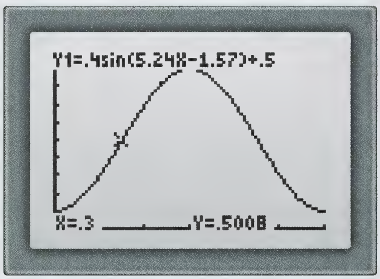
b. Use your graphing calculator to graph an equation consisting of the coefficients found in exercise 1.a. The equation is $y \doteq 0.4 \sin\left(5.24x - 1.57\right) + 0.5$.

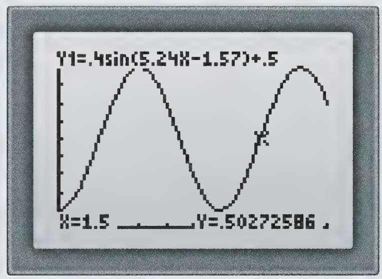
This equation describes the motion of the weight on the spring. The graphs looks as follows.





c. Use the Value feature in the CALCULATE menu to find the height at 0.3 s and at 1.5 s.

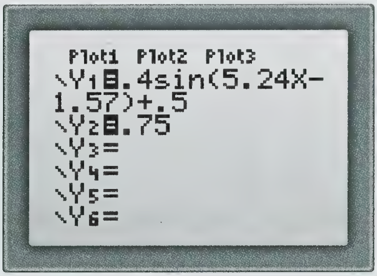


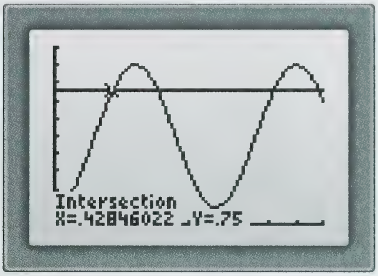


Note: To determine the value at 1.5 s, you need to extend the *x*-values in your window settings.

The height is about 0.50 m at 0.3 s and about 0.50 m at 1.5 s.

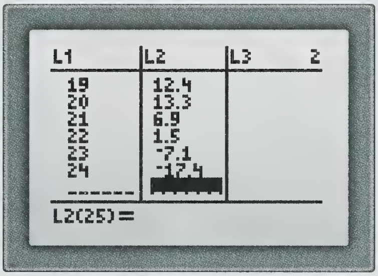
d. Add the graph of $y = 0.75$, and determine where this equation first intersects the sine equation. This is the point where the mass is 0.75 m above the tabletop for the first time.

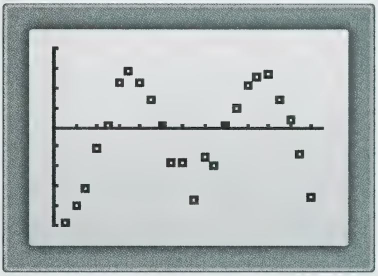




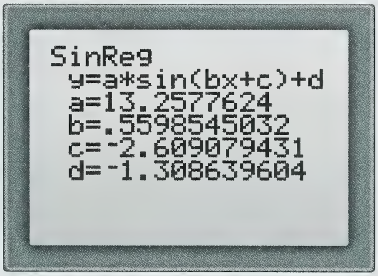
The weight will be 0.75 m above the tabletop for the first time at about 0.43 s after release.

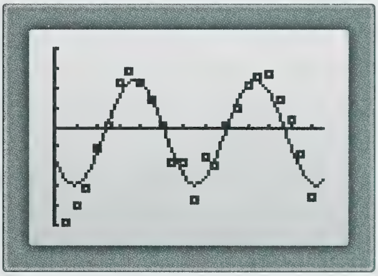
3. a. First, plot the points to find the period of the data.



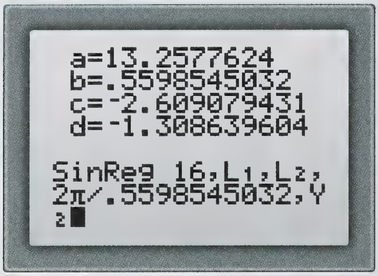


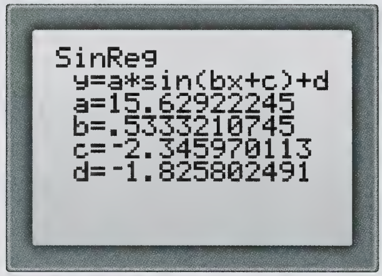
Use the default settings of SinReg $\left(\text{SinReg } L_1, L_2, Y_1\right)$.

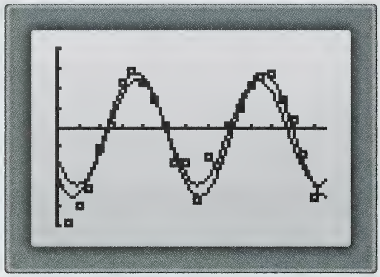




The graph doesn't seem to cover all the points well. In particular, it doesn't match the low temperatures. This means you must use a more powerful version of the regression.

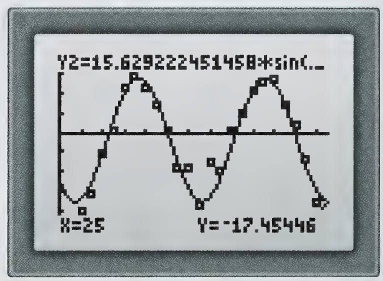






In this case, polishing the initial results gives a significantly better match.

b. January 1999 was month 25. Use the Value feature from the CALCULATE menu to find a value for month 25. **Note:** Make sure the graph extends for more than 25 months.

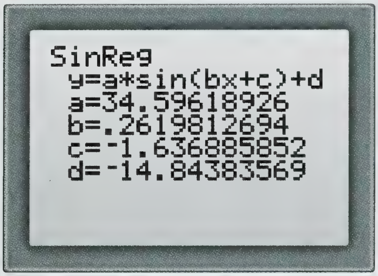


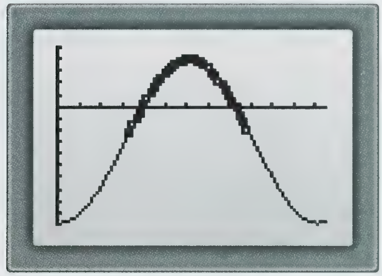
The mean minimum monthly temperature for January 1999 will be about $-17.5°\text{C}$.

c. The actual mean minimum monthly temperature for January 1999 was $-15.5°\text{C}$. This value is higher than the predicted value. The equation used to calculate the temperature was built using data for the two previous years. Weather varies tremendously from year to year. Making predictions based on very limited data is likely to give poor results. This variation also ensures that any weather prediction based on simple averages is suspect.

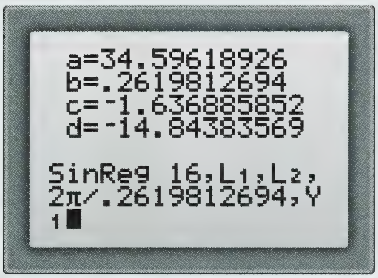
d. If the year 1999 was on average 5°C warmer than the preceding two years, you would adjust your equation upward by changing the coefficient *d* from $-1.83°\text{C}$ to 3.17°C (the result of adding 5°C to $-1.83°\text{C}$).

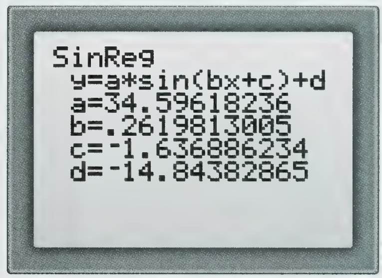
6. a. Plot the data, but remember that the period the sun rises and falls is nearly 24 hours. Use this as an initial period for this regression.

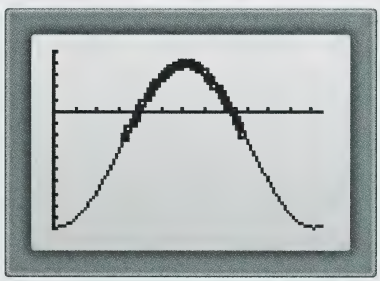




Polishing these results gives an equation that is only slightly changed.

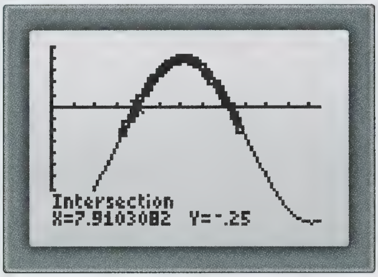


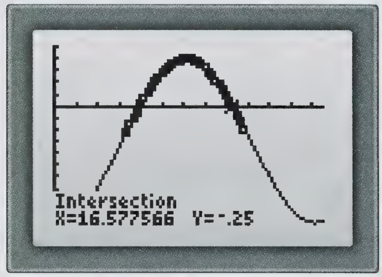


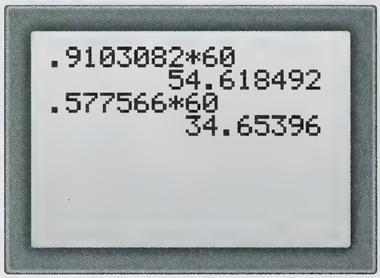


The equation of the sinusoidal curve of best fit is $y \doteq 34.60 \sin\left(0.26x - 1.64\right) - 14.84$.

b. Find the intersections of the graph and the line $y = -0.25$. This will give the times when the edge of the sun is at the horizon.







The sunrise time is about 7:55 A.M., and the sunset time is about 4:35 P.M.

c. $16.577\,566 - 7.910\,308\,2 = 8.667\,257\,8$

$60 \times 8.667\,257\,8 = 520.035\,468$

The sun is above the horizon for about 520 minutes.

d. Since the angle of elevation never reaches 90°, the sun is never directly overhead. The table of values and the regression equation both show that the maximum angle of elevation is only 19.8°.

3. Textbook exercise "Communicating the Ideas," p. 247

Answers will vary. All answers should contain a mathematical model with an explanation. A sample answer is given.

In the case of global warming, you would start by obtaining the daily temperatures in several locations around the world over a period of a few years. You would then determine if there is a pattern in the data by finding a mathematical model of the data for each location. You would then make a prediction using the mathematical model and check your prediction for the following year. If necessary, you should adjust your model to better predict the temperatures for the years to follow. You would also use your mathematical model to determine how global warming affects the temperatures in different locations. Finally, the affects of global warming on the environment can be analysed and efforts on how to reduce global warming can be initiated.

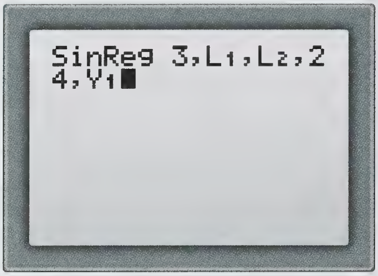
Module Review

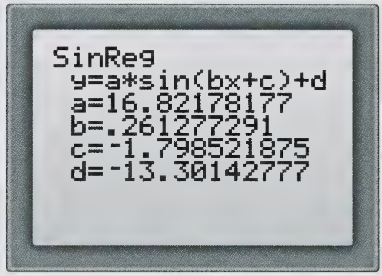
1. Textbook exercise 1 of Part A of "What Should I Be Able to Do?," p. 251

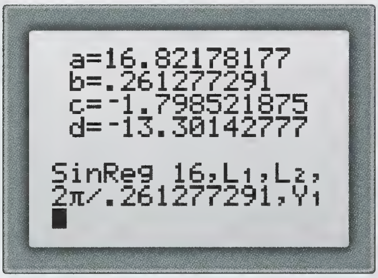
1. a.

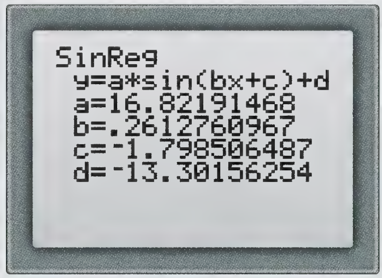
Sunrise will be between 10:00 and 10:30, approximately at 10:20. Sunset will be between 15:30 and 16:00, approximately at 15:35.

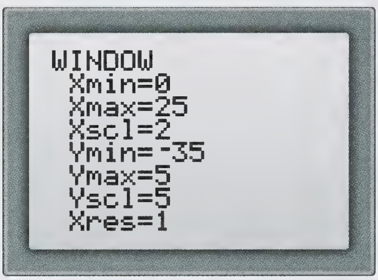
b. This data should have a period of about 24 hours.

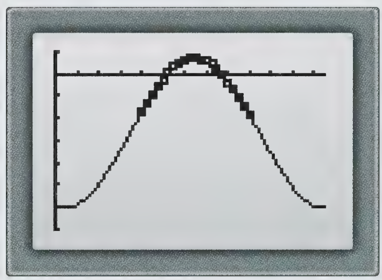






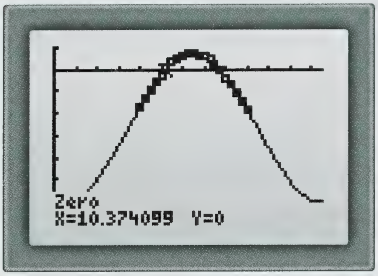


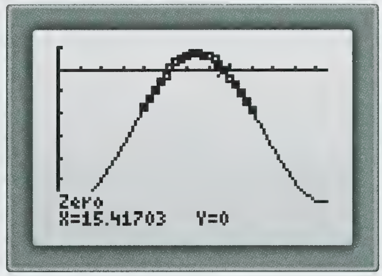


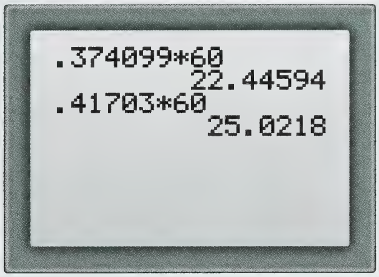


The equation of best fit is $y \doteq 16.8 \sin\left(0.261x - 1.80\right) - 13.3$.

c. Use the Zero feature from the CALCULATE menu to determine the sunrise and sunset times according to the equation.







According to the equation, the sunrise time is 10:22 and the sunset time is 15:25. These times are close to the predictions made in exercise 1.a.

d. On February 12, 2000, the sunrise in Addis Ababa, Ethiopia, was at 6:45 and the sunset was at 18:34. The sun rises much earlier and sets much later than at Nanisivik. This difference is a result of the tilt of Earth.

2. Textbook exercises 2, 4, and 6 of Part B of "What Should I Be Able to Do?," pp. 252 to 254

2. The building moves 40 cm in either direction from its normal straight position. This means the amplitude of the motion is 40 cm. Because *a* corresponds to the amplitude, *a* corresponds to the 40-cm distance.

4. a. The light seems to have a peak at 2 and a trough at 9, but the majority of the peaks are at a value of about 3.

$$\begin{aligned}\therefore \text{ Amplitude} &= \frac{9-3}{2}\\ &= 3\end{aligned}$$

The median value seems to be about 6.

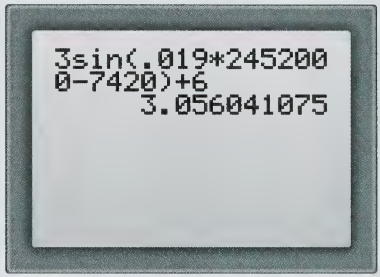
The median value at the first intersection with the curve occurs about day 2 448 920. The median value at the second intersection with the curve occurs at about day 2 449 250. Therefore, the period is around 330 days $(2\ 449\ 250 - 2\ 448\ 920)$.

b. You could estimate the equation for the intensity of Mira with an equation built from the date in exercise 4.a. The general formula for using amplitude (a), period $\left(\frac{2\pi}{b}\right)$, starting point $\left(-\frac{c}{b}, d\right)$, and median value ($d$) for periodic data is $y = a\sin\left(\frac{2\pi}{b}x + c\right) + d$. Thus, the equation for Mira would be

$$y \doteq 3\sin\left(\frac{2\pi}{330}x - \frac{2\,448\,950}{330}\right) + 6$$

This equation can be simplified to $y \doteq 3.00\sin(0.0190x - 7420) + 6.00$ (using 3 significant digits for each coefficient).

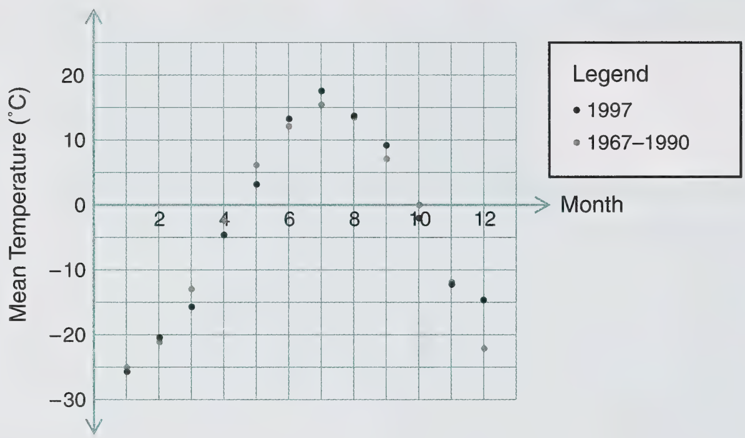
Now, determine the brightness of Mira by substituting 2 452 000 for x into the equation.



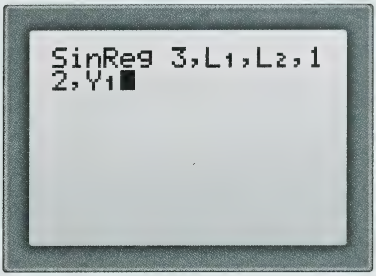
$$\begin{aligned} y &\doteq 3.00\sin(0.019x - 7420) + 6.00 \\ &\doteq 3.00\sin\left[0.019(2\,452\,000) - 7420\right] + 6.00 \\ &\doteq 3 \end{aligned}$$

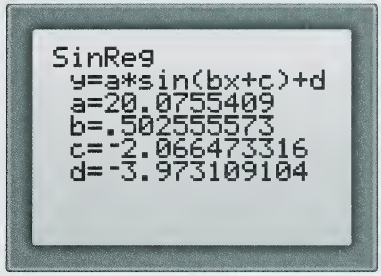
The brightness of Mira on Julian day 2 452 000 is about 3.

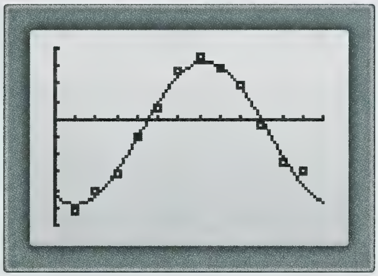
6. a.



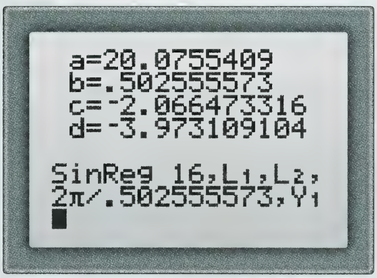
b.

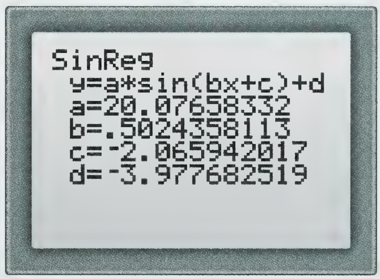


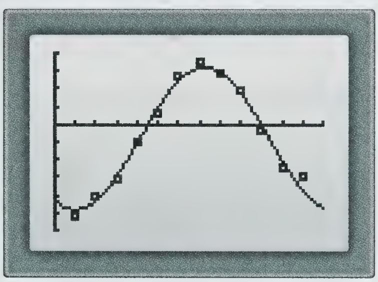




Polish this regression.

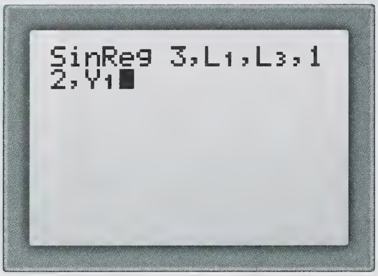


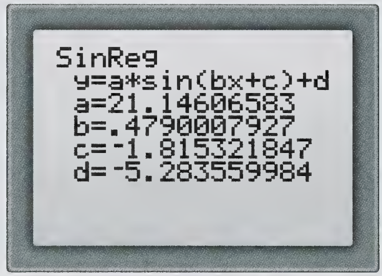


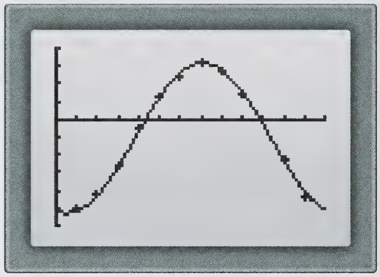


The equation of best fit is $y \doteq 20.1\sin\left(0.502x - 2.07\right) - 3.98$.

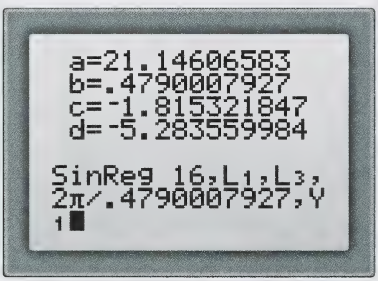
c.

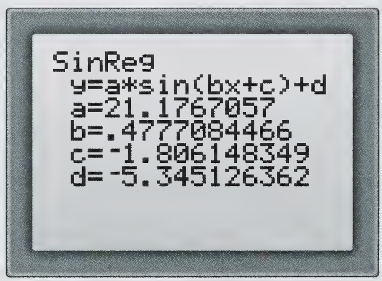


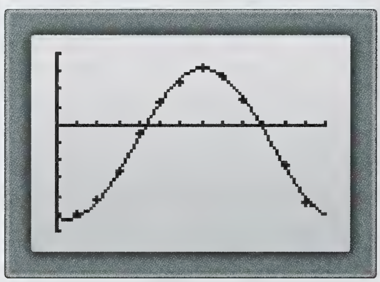




Polish this regression.

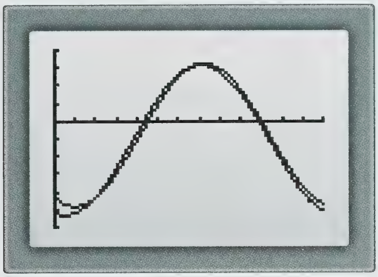






The equation of best fit is $y \doteq 21.2 \sin\left(0.478x - 1.81\right) - 5.35$.

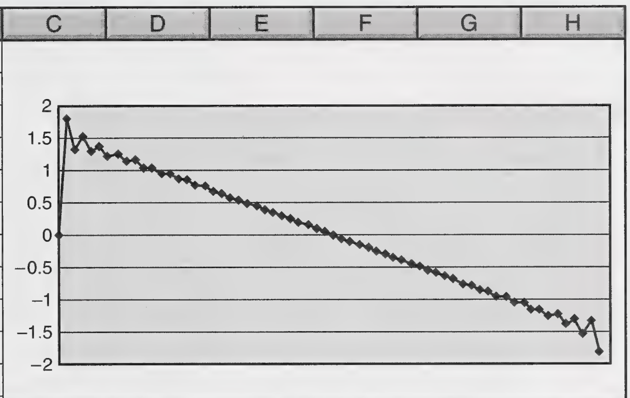
d. The equations have the same form. However, the equation of best fit for 1967–1990 has a larger amplitude, a larger period, a slightly less shifting of the starting point, and a lower median value. Therefore, because the median temperature is higher, the weather in Thompson, Manitoba, in 1997 seems to have been warmer compared to the 1967–1990 average. Here you can see how the temperatures in the range of years are lower than that for 1997 by graphing both equations of best fit on the same axes. (At least this was true for the months other than March, April, May, and June.)



Enrichment

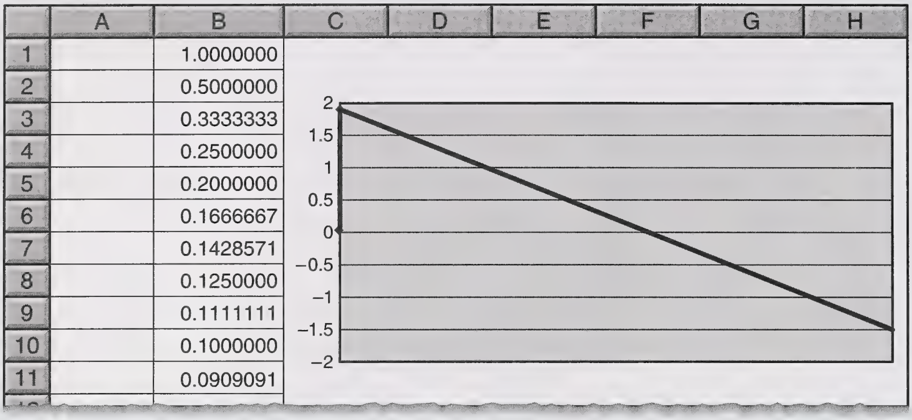
This sum of sine curves would be a triangle wave (as shown in the following graph of the sum of 31 terms).

	A	B
1		1.0000000
2		0.5000000
3		0.3333333
4		0.2500000
5		0.2000000
6		0.1666667
7		0.1428571
8		0.1250000
9		0.1111111
10		0.1000000
11		0.0909091



Module Review (continued)

The graph of the infinite sum would look like the following.



Module Project: Angle of Elevation of the Sun

Completing the Project

Textbook exercises 7, 8, and 9 of Part C of "What Should I Be Able to Do?," pp. 255 and 256

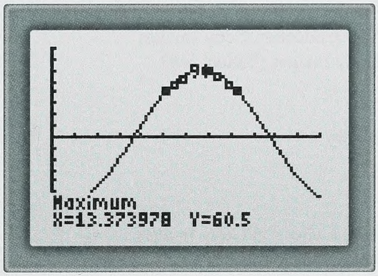
7. Errors would be expected in the measurements of time and distance. The conversions from shadow length to angle of elevation would also introduce some error. You would be better off using radian measure for the angle of elevation, since the calculator needs to be in Radian mode when performing sine regressions. Otherwise, you must make sure to change to Degree mode before finding the inverse tangent values, and then returning to Radian mode before doing the regression. The student appears to have incorrectly calculated the angle for the 3:45 P.M. measurement. It should state 46.2°, not 46.1°.

8. The maximum angle is not near 12 o'clock as you might expect. The student is taking the measurements during the part of the year when daylight savings time is in effect. This shifts the sun's time an hour ahead relative to clock time so more usable sunlight occurs in the evening hours.

   The data points are placed well. The curve drawn is perhaps a little too straight on the sides and curved too rapidly at the top, but it is quite good for a hand-drawn curve.

9. The student's work has a major problem. The equation of best fit doesn't match reality. The elevation of the sun follows a 24-hour cycle, not the 8-hour cycle the chosen equation of best fit gives. This might be an example of not giving the calculator an estimated initial period of the data.

An equation like $y \doteq 59.5 \sin\left(0.262x + 4.35\right) + 1.00$ fits the data quite closely and keeps the 24-hour period that should be expected. (The maximum value of this equation occurs at 60.5.)



The table in Example 3 is actually found on page 243, not page 277. The student used 22.4 for June 3, but this is for June 4 according to the table.

Using the equation and data that the student used, the calculation of latitude is carried out correctly.

The percentage error in the latitude would be a function of measurement errors, regression errors, and errors in estimating the maximum elevation. The worst time-measurement error (assuming a correctly functioning clock) would be 0.5 min. This would lead to a maximum time error of about $\pm 1\%$. The maximum measuring error of length would be 0.25 cm. This would lead to a maximum error here of about $\pm 0.4\%$. Taking an inverse tangent will introduce some additional error, but it should be small (say $\pm 0.1\%$). The angles of elevation should then be accurate to within $\pm 1.5\%$. Finding a curve of best fit will introduce error as well. Let's say this error will be about $\pm 1\%$. (A calculated maximum value of 60.8 shows that there is some error but not really much.) The total error would then be $\pm 4.0\%$.

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